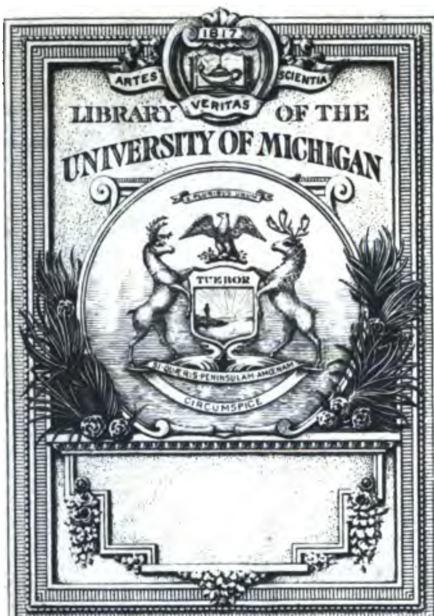
1817
ARTES
SCIENTIA
VERITAS
LIBRARY OF THE
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SI QUÆRIS PENINSULAM AMŒNAM
CIRCUMSPICE

x = [illegible]

$24 - x = \frac{x^2}{100}$

$2400 - 100x = x^2$

$x^2 + 100x = 2400$

$x^2 + 100x + 2500 = 4900$

$x + 50 = 70$

$x = 70 - 50 = 20$

ARITHMETICAL SPYGLASS

AND

TEACHER'S ASSISTANT,

INTENDED AS A

KEY AND SUPPLEMENT

TO THE

DIFFERENT WORKS ON ARITHMETIC.

FOR THE USE OF

SCHOOLS AND ACADEMIES.

BY CHARLES WATERHOUSE,

TEACHER OF MATHEMATICS.

SECOND EDITION, REVISED AND ENLARGED.

PORTLAND.

WATERHOUSE & CO.—WM. HYDE.—R. H. SHERBURNE.
DAY, LYON & CO.—H. J. LITTLE & CO.—
S. H. COLESWORTHY.

1842.

PRINTED AT THE AMERICAN OFFICE.
NO. 1, UNION STREET, PORTLAND.

PREFACE.

It is well known that common works on Arithmetic abound with curious and abstruse questions more perplexing than beneficial, but if *partially* explained serve as an inducement rather than an obstacle to the study so solitary in its character.

These questions the inquisitive and ingenious student is anxious to solve, and after puzzling a while, applies to the teacher for assistance.

To teachers, these applications are necessarily too great a tax on their time, especially when having a large number of pupils under their care.

Besides, in the hurry of business, it is often very difficult for persons of good retentive abilities, that have attained to the knowledge of Mathematics, not to be expected of our primary school teachers, to recollect, at the moment, all the principles by which are solved some of the difficult questions that may be taken from any of the many works on arithmetic; the judgment of some to the contrary notwithstanding.

Even the Professor in College, who is confined to one or two branches, on which his powers are concentrated, makes free use of his translations, and if these are useful to him, certainly similar helps ought not to be denied the common teacher, who has several branches to look after.

And the various studies taught in our common schools with the difficulties that teachers labor under, forbid the idea of illustrating the principles of intricate questions, without being partial in the application of time or energies—And pupils by not being thus furnished, lose the knowledge of Mathematical principles they should have for assistance through life, and never obtain what they might have for a convenience.

Sometimes also, questions are found in works containing no rule or precedent for solving them.

Likewise, difficult questions are much used by *wiseacres* and the *ill-disposed* to annoy teachers and injure schools by engaging the attention of the teacher which might be devoted, at particular times, to more useful purposes.

Hence, whatever tends to promote the usefulness of the instructor, must commend itself to all.

Also, it is believed that every judicious attempt to facilitate the study of a science so essential to the interests of all, will be received with indulgence by an enlightened community, and meet the entire approbation of a generous public.

When considering the preceding facts, and in order to give advanced scholars the privilege of extending their knowledge to a full development of the elementary principles in the higher operations of Arithmetic; also with the view of enabling teachers to lay demonstrations of the dogmatical rules, and an explanation of the abstruse matter before their class studying this science, I composed this Desideratum.

It is presumed that it would be studied to good advantage, in connexion with other works on Arithmetic, by the *Teachers' class in Academies.*

Particular attention has been paid to render it concise and free from those mistakes which are commonly to be met with in our various text-books on Arithmetic. Therefore, it is hoped that if the tongue of the captious caviler should blazon defects, for which others might search in vain, that the eye of the candid critic will not see objections in it which reason and truth would long hesitate to approve.

It is not presumed however, that it is without imperfections, the same at least as may always be met with in any and all the works of man. In this production, although availed of the best Authors which could be obtained, I followed none particularly, except Bonnycastle's method of demonstration. And every item deemed intricate, rare, useful, and interesting, occupies its proper place in this work, and matter considered

of a superfluous and common nature was carefully excluded.

Since the utility of a work like the present, if faithfully prepared, is so obvious as to require no comment, its author solicits the candid consideration of the public in presenting it, with full confidence that it will serve to lessen the labor of instructors and facilitate the improvement of their pupils.

THE AUTHOR.

N. Pownal (Me.) A. D. 1842.

This treatise may be considered as an Appendix to the works of the following authors on Arithmetic, and as containing precedents for the solution of their difficult questions, viz.: Messrs. WELCH, WALSH, SMITH, PIKE, BEECHER, KENNE, HALL, DABOLL, LEAVITT, COLBURN, ROBINSON, BURNHAM, ROSE, ADAMS, STANIFORD, OLNEY, LEONARD, ROOT, EMERSON, BAILEY, TRACY, DAVIES, BONNEYCASTLE, TYLER, and GREENLEAF.

C. W.

TEACHER'S

AND

ADVANCED SCHOLAR'S

ARITHMETIC.

ARITHMETICAL DEVELOPMENTS.

I. Arithmetic is the Science and Art for comprehending Multitude, and also Magnitude, in some degree.

Its theory is a Science. Its application to business, is an Art.

The *theory* admits of but three fundamental divisions, viz.: Numeration, Addition, and Subtraction.

Multiplication and Division are short methods of performing many additions and subtractions: consequently, not fundamental principles, but rules that emanate from, and are *attributes* of Addition and Subtraction.

II. NUMERATION.

The characters used to express numbers, are called *figures*, and have two values, viz.: *simple* and *local*.*

A figure standing alone, or at the right hand of other figures, means as many units as practice has made it to represent: therefore, its value is called *simple*, and any figure otherwise situated, has a *local* value.

Numerating from *right* to *left* is merely arbitrary.

Numerating from *left* to *right*, except for convenience, is just as proper.

Every left-hand place, in numerating from right to left, is as many times larger than its right-hand place as the ratio used in such numeration implies.

The custom of using the ten-fold ratio in arithmetical com-

* *Local* is from the Latin word *locus*, which means *place*, or value according to the place occupied.

putations, though the most convenient of any, is entirely arbitrary, as any ratio might be adopted, by giving it a corresponding number of characters, one of which should be a cipher, or *two* of the different characters would express the some value, and thereby cause confusion.

The ten-fold ratio being adopted, it follows that every right-hand place is but one-tenth the value of its left-hand place, *ad infinitum*,* without any respect to the point used to separate decimals from whole numbers: consequently, the adoption of the *separatrix*, was only for distinguishing the starting point of decimals, or whole numbers.

FIGURES are divided into periods of six places each, and the first part of any period, is so many *units* of it, and the latter part so many *thousands*.

NUMERATION TABLE.

8	Hund. of Thou. of Sextillions.	9	Hund. of Thou. of Tredecillions.
8	Tens of " of "	9	Tens of " of "
8,	- - " of "	9,	- - " of "
9	Hund. of - - "	8	Hund. of - - "
9	Tens of - - - "	8	Tens of - - - "
9;	- - - - - "	8;	- - - - - "
9	Hund. of Thou. of Quintillions.	7	Hund. of Thou. of Duodecillions.
9	Tens of " of "	7	Tens of " of "
9,	- - " of "	7,	- - " of "
8	Hund. of - - "	6	Hund. of - - "
8	Tens of - - - "	6	Tens of - - - "
8;	- - - - - "	6;	- - - - - "
7	Hund. of Thou. of Quartrillions.	5	Hund. of Thou. of Undecillions.
7	Tens of " of "	5	Tens of " of "
7,	- - " of "	5,	- - " of "
6	Hund. of - - "	4	Hund. of - - "
6	Tens of - - - "	4	Tens of - - - "
6;	- - - - - "	4;	- - - - - "
5	Hund. of Thou. of Trillions.	3	Hund. of Thou. of Decillions.
5	Tens of " of "	3	Tens of " of "
5,	- - " of "	3,	- - " of "
4	Hund. of - - "	2	Hund. of - - "
4	Tens of - - - "	2	Tens of - - - "
4;	- - - - - "	2;	- - - - - "
3	Hund. of Thou. of Billions.	2	Hund. of Thou. of Nonillions.
3	Tens of " of "	2	Tens of " of "
3,	- - " of "	2,	- - " of "
2	Hund. of - - "	3	Hund. of - - "
2	Tens of - - - "	3	Tens of - - - "
2;	- - - - - "	3;	- - - - - "
2	Hund. of Thou. of Millions.	4	Hund. of Thou. of Octillions.
2	Tens of " of "	4	Tens of " of "
2,	- " of - "	4,	- - " of "
3	Hund. of - - - "	5	Hund. of - - "
3	Tens of - - - "	5	Tens of - - - "
3;	- - - - - "	5;	- - - - - "
4	Hundreds of Thousands.	6	Hund. of Thou. of Septillions.
4	Tens of Thousands.	6	Tens of " of "
4,	Thousands.	6,	- - " of "
5	Hundreds.	7	Hund. of - - "
5	Tens.	7	Tens of - - - "
5	Units.	7	- - - - - "

* *Ad Infinitum* is a Latin word, which signifies *to endless extent.*

III. ADDITION.

DEMONSTRATION OF THE RULE.

This rule is founded on the known axiom—"*The whole is equal to the sum of all its parts.*" It has been shown that Numeration reduces every number to a certain number of places or orders, each of which is tenfold less in value than its preceding place: consequently, one is carried for every ten, in Addition, because *common consent* adopts the tenfold ratio in Arithmetical computations.

To illustrate the preceding axiom, with perspicuity, let the numbers, 437, 258, 345, and 653, be decomposed, and their several parts be added, as in the margin.

Operation.

The first is	400 +	30 +	7 =	437
The second is	200 +	50 +	8 =	258
The third is	300 +	40 +	5 =	345
The fourth is	600 +	50 +	3 =	653
The whole is	1500 +	170 +	23 =	1693

Explanation.—Here it will be seen that the column of units amounts to 23 *units*. The column of tens amounts to 170 *units*, or 17 *tens*. The column of hundreds amounts to 1500 *units*, 150 *tens*, or 15 *hundreds*.

But to save the trouble of setting down and adding up so many separate amounts, a way has been contrived of carrying the left hand figure immediately, and uniting it with the next column; from which the Rule is derived.

TO PROVE ADDITION.

Rule.—Set the excess of nines in each row of figures to the right of its row, and if the excess of nines in the sums result, and the column made by setting out the several excesses, are alike, the work is right.

Example.

9876	3
8765	8
7654	4
26295	6

Remark.—The figure 9 has a peculiar property, which, except 3, belongs to no other figure whatever, viz: *that any number divided by* 9 *will leave the same remainder as the sum of its figures divided by* 9.

Note 1.—Compound Addition differs from Simple Addition only in the succession of its orders: consequently, to the ingenious mind, its demonstration seems to be unnecessary.

Note 2.—Multiplication being in substance Addition, the remarks just made respecting Compound Addition are applicable to Compound Multiplication.

IV. SUBTRACTION.

DEMONSTRATION OF THE RULE.

1. When all the places of the least number are less than their correspondent places in the larger, the difference of the figures in the several like places, must, taken together, make the true difference sought; because, "*as the sum of the parts is equal to the whole*," *so is* "*the sum of the differences, of all the similar parts, equal to the differences of the whole.*"

2. Borrowing from a preceding place to increase an upper place when its correspondent lower place is largest, is only resolving the upper number into such parts, as are, each, greater than, or equal to, the similar parts of the less number.

EXAMPLE. From 76254 take 28786.

Let the numbers be decomposed and arranged as in the margin.

Operation.

$$\begin{array}{l} 70000+6000+200+50+4=76254 \\ 20000+8000+700+80+6=28786 \\ \hline 40000+7000+400+60+8=47468 \end{array}$$

EXPLANATION.—Here I begin at the right hand, and finding that I cannot take 6 from 4, I borrow 10 from 50, and add it to 4, which makes 14. From this I take 6 and set down 8. As 10 is borrowed from 50, there is 40 left. I cannot take 80 from 40; I therefore borrow 100 from 200, and add it to 40, making 140; from which I take 80, and set down 60; and so on through the whole.

In borrowing to add to an upper figure, or place, instead of considering the next upper figure, or place, diminished, it has been found most convenient to increase the next lower figure, or place, which brings the result just the same.

On this principle was founded the Rule for Subtraction.

TO PROVE SUBTRACTION.

RULE. 1.—Having subtracted as usual, cast out the 9s from the minuend, and place the excess at the right hand.

2. Cast out the nines from the subtrahend and remainder, and add their excesses together; and if the work is right, the excess of 9s in their sum will be the same as the excess of 9s in the minuend.

Example.

From 46875 subtract 34789.

Operation.

$$\begin{array}{lcl} 46875 & - - - & 3,\ \text{excess.} \\ 34789 & - - - & \left.\begin{array}{l} 4, \\ \\ 8, \end{array}\right\}\ \text{excesses.} \\ \cline{1-1} 12086 & & \\ & & \overline{12},\ \text{sum.} \\ & & \overline{3},\ \text{excess.} \end{array}$$

Remark.—As the subtrahend and remainder form a sum in Addition, of which the minuend is the amount, the *reason* of the *proof* is obvious.

Note 1.—Compound Subtraction differs from Simple Subtraction only in the succession of its orders: consequently, its demonstration seems to be unnecessary.

Note 2.—Division being in substance Subtraction, the remarks just made respecting Compound Subtraction are applicable to Compound Division.

V. MULTIPLICATION.

DEMONSTRATION OF THE RULE.

1. When the multiplier is a single digit, it is plain that we find the product; for by multiplying *every part* of the multiplicand, it is evident we multiply *the whole*; and in writing down the products, which are less than ten, or the excess of tens, under the place of the figures multiplied, and carrying the *tens* to the product of the next place, is only gathering together the similar parts of the respective products, and is therefore the same in effect, as though we wrote down the multiplicand as often as the multiplier expresses, and added them together; for the sum of every column is the product of the figures in the place of that column and the products, collected together are evidently equal to the whole required product.

2. When the multiplier consists of several figures, we find the product of the multiplicand by the unit figure, and then suppose the multiplier divided into parts, and, after the same manner, find the product of the multiplicand by the second figure of the multiplier; but, as the figure by which we are multiplying, stands in the place of tens, the product must be ten times its simple value; and, therefore, the first figure in this product, must be noted in the place of tens, or, which is the same, directly under the figure we are multiplying by. And proceeding in the same manner with all the figures in the multiplier, separately, it is evident we shall multiply all the parts of the multiplicand by all the parts of the multiplier; therefore, these several products being added together, will be equal to the whole required product.

3. The reason of the method of *proof*, depends upon this proposition, *that if two numbers are to be multiplied together,*

either of them may be made the multiplier or multiplicand, and the product will be the same.

EXAMPLE.—In order to illustrate the demonstration, let 568 be multiplied by 476.

These numbers may be decomposed and multiplied thus:—

OPERATION.

Multiplicand,	500	+ 60	+ 8	=	568
Multiplier,	400	+ 70	+ 6	=	476
	3000	+ 360	+ 48	=	3408
3500+	4200	+ 560		=	3976
200000+24000+	3200			=	2272
200000+59000+	10400	+ 920	+ 48	=	270368

EXPLANATION.

You see, in the above process, we multiplied through, first by 6 *units*, then through by 7 *tens*, or 70, and then by 4 *hundreds*, or 400, placing the several products underneath, and adding them up.

Lastly, the sums of these products are added, making 270368, the total product.

The preceding shews, that the multiplicand is taken as many times as there are units in the multiplier.

On this principle, was founded the Rule of Multiplication.

REMARK.—Multiplication may be proved by casting out the 9s; but is liable to this inconvenience, viz.: *The work will always prove right when it is so; but it will not always be right when it proves so.*

BRIEF METHODS OF MULTIPLYING.

1. *When the multiplier is any number of 9s.*

RULE.—To the right of the multiplicand write as many 0s as there are 9s in the multiplier—under this new multiplicand write the *given* one, units, &c. under units, &c.—then subtract, and this difference is just the same as if the general method had been pursued.

Operation.

EXAMPLE.—Multiply 987654 by 999. { 987654000
987654

Their product is - - - - - 986666346

Reason. If a number be multiplied by 9, the product is but nine-tenths of the product of the same sum, multiplied by 10; and, as annexing a cypher, to the right hand of the multiplicand, supposes it to be increased tenfold; therefore, subtracting the given multiplicand from the tenfold multiplicand, it is evident that the remainder will be ninefold the given multiplicand, and equal to the product of the same by 9; this will hold true of any number of nines, and this principle may be extended to other numbers.

2. *When the multiplier is* 13, 14, &c. *to* 19.

Rule.—Place the multiplier at the right of the multiplicand, with the sign of multiplication between them,—multiply the multiplicand by the unit figure of the multiplier, removing the product one place to the right of the multiplicand; this product and the multiplicand make the total product.

Example.

75964×13
227892
———
987532

3. *When the multiplier is* 101, 102, &c. *to* 109.

Rule. Multiply by the unit figure of the multiplier, remove the product two places to the right of the multiplicand—add together as before for the product.

4. *When the multiplier is* 111, 112, 113, &c. *to* 119.

Rule. Multiply by the unit figure only of the multiplier, and add to each multiplication the two figures, which stand next at the right hand of that which is multiplied, and to the two last figures, *separately*, add what you carry.

Example.

9417
119
———
1120623

5. *When the multiplier is* 21, 31, &c. *to* 91.

Rule.—Multiply by the ten's figure, only, of the multiplier; and set the unit figure of the product under the place of tens; add them all together, and their sum is the total product.

6. *When the multiplier is* 22, 23, &c. *to* 29.

Rule.—Multiply every figure of the multiplicand by the unit figure of the multiplier, and add to each product twice that figure which stands next at the right hand of the figure, you multiplied; and to twice the last figure add what you carry.

Example.

7657
29
———
222053

A TABLE OF SQUARES.

21×21=441	24×24=576	27×27=729
22×22=484	25×25=625	28×28=784
23×23=529	26×26=676	29×29=841

A MULTIPLICATION TABLE.

	13	14	15	16	17	18	19
4	52	56	60	64	68	72	76
5	65	70	75	80	85	90	95
6	78	84	90	96	102	108	114
7	91	98	105	112	119	126	133
8	104	112	120	128	136	144	152
9	117	126	135	144	153	162	171
11	143	154	165	176	187	198	209
12	156	168	180	192	204	216	228
13	169	182	195	208	221	234	247
14	182	196	210	224	238	252	266
15	195	210	225	240	255	270	285
16	208	224	240	256	272	288	304
17	221	238	255	272	289	306	323
18	234	252	270	288	306	324	342
19	247	266	285	304	323	342	361

VI. DIVISION.

DEMONSTRATION OF THE RULE.

According to the rule, we resolve the dividend into parts, and find, by trial, the number of times the divisor is contained in each of those parts: consequently, to illustrate, let 8686, divided by 43, be separated and arranged as follows:

OPERATION.

Divisor.	Dividend.	Quotient.
$40+3$)	$8000+600+80+6$ (	$200+2=202$,true quotient.
	$8000+600$	
	$80+6$	
	$80+6$	

Explanation.—I find first, that, 40 is contained in 8000, 200; then I multiply the whole divisor, $(40+3)$ by 200, which makes $8000+600$. These I put under the first two

terms of the dividend, and subtract, and nothing remains. I then bring down the other two terms and proceed in the same manner.

Here it will be seen that the first quotient figure, taken in its complete value from the place it stands in, is the true quotient of the divisor, in the complete value of the first part of the dividend.

For the same reason all the rest of the figures of the quotient, taken according to their places, are, each, the true quotient of the divisor, in the complete value of the several parts of the dividend belonging to each: consequently, all the quotient figures, taken in order, is the true quotient of the whole dividend by the divisor.

NOTE.—Division being the converse of Multiplication, the same difficulty attending the proving of Multiplication, "by casting out the nines," is also liable to Division.

TO PROVE DIVISION.

RULE.—Add the remainder and all the products of the several quotient figures multiplied by the divisor together, according to the order in which they stand in the work, and the sum, with the remainder, if any, when the work is right, will be equal to the dividend.

Example.

```
79)987654(12501
   79=Divisor×1
   ---
   197
   158=  "   ×2
   ---
    396
    395=  "   ×5
    ---
      154
       79="   ×1
       --
       75= Rem.
  ---------------
   987654, Proof.
```

VII. PROPERTIES OF NUMBERS.

1. The product of an even, and an odd number, or of two even numbers, is even.
2. The product of any two odd numbers is an odd number.
3. If an odd number measure an even number, it will also measure the half of it.
4. The difference between an integral cube and its root, is always divisible by 6.

5. The product arising from two different prime numbers cannot be a square.

6. A prime number is that which can only be measured by unity.

7. The product of no two different numbers, prime to each other, can make a square, unless each of those numbers be a square.

8. Every prime number above 2, is either 1 greater or 1 less than some multiple of 4.

9. Every prime number above 3, is either one greater or 1 less than some multiple of 6.

10. The number of prime numbers is unlimited. The first ten are, 1, 2, 3, 5, 7, 11, 13, 17, 19, 23.

11. One number is prime to another, when unity is the only number by which both can be measured.

12. If equal quantities be added to, subtracted from, multiplied or divided by equal quantities, the wholes, remainders, products and quotients will be respectively equal.

13. Two quantities respectively equal to a third, are equal to each other.

14. The equal powers or roots of equal quantities are equal.

15. A *perfect number* is equal to the sum of all its aliquot parts. Thus, 6=3×2×1, here 3,2 or 1, or all, will divide 6.

VIII. TABLES OF WEIGHTS AND MEASURES.

TROY WEIGHT.

	Grains.
Pennyweight, 1=	24
Ounce, 1= 20=	480
Pound, 1=12=240=	5760

APOTHECARIES' WEIGHT.

	Grains.
Scruple, 1=	20
Dram, 1= 3=	60
Ounce, 1= 8= 24=	480
Pound, 1=12=96=288=	5760

NOTE.—175 oz. Troy is 192 oz. Avoirdupois.

REFINERS' WEIGHT.

	Blanks.
Perrot, 1=	20
Mite, 1= 20=	480
Grain, 1=20=400=	9600

NOTE.—The Carat is a 24th part of gold or silver.

PLANETARY MOTION.

				Seconds.
			Minute, 1=	60
		Degree, 1=	60=	3600
	Sign, 1=	30=	1800=	108000
Zodiac, =	12=	360=	21600=	1296000

AVOIRDUPOIS WEIGHT.

					Drams.
				Ounce, 1=	16
			Pound, 1=	16=	258
		Quarter, 1=	28=	448=	7168
	Hundred, 1=	4=	112=	1792=	28672
Ton, 1=	20=	80=	2240=	35840=	573440

TIME.

						Seconds.
					Minute, 1=	60
				Hour, 1=	60=	3600
			Day, 1=	24=	1440=	86400
		Week, 1=	7=	168=	10080=	604800
	Month, 1=	4=	28=	672=	40320=	2419200
Julian Y'r, 1=52w. 1d. 6h.=			$365\frac{1}{4}$=	8766=	525960=	31557600
Periodical Y'r, 1=$14\frac{1}{2}$s. 9m.			$365\frac{1}{4}$=	8766=	525969=	31558154
Tropical Y'r, 1=57s. 48m.			$365\frac{5}{24}$=	8765=	525948=	31556937

NOTE.—The twelve calendar months has each, the following number of days, viz.

The fourth, eleventh, ninth and sixth
Have thirty days to each affixed;
All the rest have thirty-one,
Except the second month alone,
Which hath twenty-eight, in fine,
Till leap-year gives it twenty-nine.

LONG MEASURE.

					Barley Corns.
				Inch, 1=	3
			Foot, 1=	12=	36
		Yard, 1=	3=	36=	108
	Rod, 1=	$5\frac{1}{2}$=	$16\frac{1}{2}$=	198=	594
Furlong, 1=	40=	220=	660=	7920=	23760
Mile, 1=8=	320=	1760=	5280=	63360=	190080

NOTE.—$7\frac{92}{100}$in. is 1 link; 25 links is 1 Rod.

SQUARE MEASURE.

						Inches.
					Foot, 1=	144
				Yard, 1=	9=	1296
			Rod, 1=	$30\frac{1}{4}$=	$272\frac{1}{4}$=	39204
		Rood, 1=	40=	1210=	10890=	1568160
	Acre, 1=	4=	160=	4840=	43560=	6272640
Mile, 1=	640=	2560=	102400=	3097600=	27878400=	4014489600

SOLID MEASURE.

		Inches.
	Foot, 1=	1728
Yard, 1=	27=	46656

NOTE 1.—16 cubic feet is 1 foot of wood, and 8 ft. of wood is 1 cord.

NOTE 2.—All carpenters in the United States, allow 40 cubic feet of *hewn* timber to make a ton; consequently, 50 cubic feet of *round* timber is a ton; yet in every Arithmetic we find it thus: 50 cubic feet of hewn, &c. make a ton.

This error, without doubt, was originally, a typographical one, but faithfully copied by every subsequent author; and I hardly know which to wonder at the most,—why the same error should pass uncorrected through so many able hands, or that the copiers should blunder so prodigiously over one another.

WINE MEASURE.

							Cubic Inches.
						Pint, 1=	$28\frac{7}{8}$
					Quart, 1=	2=	$57\frac{3}{4}$
				Gallon, 1=	4=	8=	231
			Tierce, 1=	42=	168=	336=	9702
		Hogshead, 1=	$1\frac{1}{2}$=	63=	252=	504=	14553
	Puncheon, 1=	$1\frac{1}{3}$=	2=	84=	336=	672=	19404
Pipe, 1=	$1\frac{1}{2}$=	2=	3=	126=	504=	1008=	29106
Tun, 1=2=	3=	4=	6=	252=	1008=	2016=	58212

DRY MEASURE.

			Gallons.	Cubic Inches.
		Peck, 1=	2=	$537\frac{3}{5}$
	Bushel, 1=	4=	8=	$2150\frac{2}{5}$
Quarter, 1=	8=	32=	64=	$17203\frac{1}{5}$
Wey, 1=	40=	160=	320=	86016
Last, 1=	80=	320=	640=	172032

NOTE 1.—A Chaldron, (U. S.)=32 bushels.

NOTE 2.—A gallon dry measure is $268\frac{4}{5}$ cubic inches.

ALE MEASURE.

	Gallons.
Firkin,	1= 8
Kilderkin,	1=2=16
Barrels,	1=2=4=32
Hogshead,	1=1½=3=6=48

BEER MEASURE.

Gallons.	
9=	1 Firkin.
18=	2=1 Kilderkin.
36=	4=2=1 Barrel.
54=	6=3=1½=1 Hhd.
72=	8=4=2=1 Punch'n.
108=	12=6=3=2=1 Butt.

NOTE.—A gallon, beer or ale measure, is 282 cubic inches.

IX. PROBLEMS,

WITH RULES, AND QUESTIONS TO ILLUSTRATE THE RULES.

DEFINITION.

A Problem is a proposition or a question requiring something to be done; either to investigate some truth or property, or to perform some operation.

PROB. I.—*The sum of two numbers, and the difference of their squares given, to find those numbers.*

RULE 1.—Divide the difference of their squares by the sum of the numbers, and the quotient will be their difference.

2. Subtract the difference from the sum, and half the remainder will be the smaller number. Then add the difference to the smaller number and you have the larger number.

EXAMPLES.

1. A and B played at marbles, having 14 apiece at first; but after playing several games, B having lost some of his, would not play any longer, and it was found that the difference of the squares of what each then had was 336. How many did B lose?

Thus, $\overline{14+14})336(12$ diff.; 14=half sum, and $12\div2=6$, half diff.

Then $14+6=20$, A retired with. And $14-6=8$ B had left; then, $14-8=6$ marbles that B lost.

2. What are the two fractions whose sum is $\frac{43}{50}$, and whose difference is $\frac{6}{7}$? Questions of this nature are solved by the latter clause of the preceding Rule. Thus $\frac{43}{50}-\frac{6}{7}=\frac{287}{450}$, then $\frac{287}{450}\div\frac{4}{2}=\frac{287}{900}$, less fraction, and $\frac{287}{900}+\frac{6}{7}=\frac{487}{900}$, greater fraction.

PROB. II. *The difference of two numbers, and the difference of their squares given, to find those numbers.*

RULE.—Divide the difference of their squares by the difference of their numbers, and the quotient will be their sum. You then have their sum and difference to proceed by PROB. I.

EXAMPLE.

Said Ai to Lorenzo, Father gave me \$12 more than he gave Charles, and the difference of the squares of our separate parcels is 288. How much did he give each. ?

Thus, $288 \div 12 = 24$, the sum; then $\overline{24-12} \div 2 = 6$; then $12+6=$ \$18, Ai's share; $12-6=$ \$6, Charles had given him.

PROB. III.—*The product of three factors, and two of those factors given, to find the third factor.*

RULE.—Divide the given product by the product of the two known factors, and the quotient is the factor required.

EXAMPLES.

1. What must be the length of a stick of *hewn* timber that is 10 inches wide, and 1 ft. 3 in. deep, in order to contain 1 ton?

First, consider 1 ton the product of 3 factors. Then, 40 cubic feet=1 ton=69120 in.; and 10 in.$\times$1 ft. 3 in.=150 in. Then, $\underline{69120 \div 150} \div 12 = 38\frac{2}{5}$ feet in length, Ans.

2. Suppose wood to be piled on a base 15 ft. 6 in. long, and 7 ft. 9 in. wide, what must be the height of the pile to contain 16 cords?

First, 16 cords=2048 solid feet, the product of 3 factors; and 15 feet 6. in.$\times$7ft. 9in.$=120\frac{1}{8}$ft., the product of the 2 given factors. Then $2048 \div 120\frac{1}{8} = 17{,}048907+$ft. in height, the required factor.

PROB. IV.—*To find a divisor that will divide two or more numbers without a remainder.*

RULE 1.—Divide the larger number by the smaller, and this divisor by the remainder, and thus continue dividing the last divisor by the last remainder till nothing remains, and the last divisor is the divisor sought, if only two numbers are given :—

2. But if more than two numbers are given, first find a divisor for any two of the numbers; then find a divisor to the found divisor and another of the given numbers, and thus proceed till all the given numbers are brought in, *and the last divisor used is the divisor sought.*

EXAMPLES.

1. Suppose a hall to be 154 feet long and 55 feet wide, what is the length of the longest pole that will exactly measure both the length and width of said hall?

Thus, $154 \div 55 = 2, + 44$ rem.; then $55 \div 44 = 1$, $+ 11$ rem.; again, $44 \div 11 = 4$; consequently, a 11 foot pole will be the answer.

2. A owns 720, B 336, and C 1736 rods of land. They agree to divide it into equal house lots, fixing on the greatest number of rods for a lot, that will allow each owner to lay out all his land. How many rods must there be in a lot, how many lots has each, and how many lots in all?

First.

336)720(2
 672
 ——
 48)336(7

Then, 48)1736(3
 1728
 ——
Last divisor, 8)48(6

Therefore 8 rods in each lot,

Lastly.

8	720	90=A's
	336	42=B's
	1736	217=C's

And 349 lots in all.

PROB. V.—*To find the dividend that will contain two or more numbers given, without a remainder, when either of the given numbers is used as a divisor.*

RULE.—Call the numbers the denominators of so many fractions, and a common denominator found thereto, in the manner prescribed for finding a common denominator to vugar fractions, is the dividend sought.

EXAMPLE.

Allowing 63 gallons to fill a hogshead, 42 a tierce, and 32 a barrel, what is the smallest quantity of molasses that can be first shipped in some number of full hogsheads, then discharged and re-shipped in some number of full tierces, and again discharged and re-shipped in some number of full barrels.

Operation.

3	63	42	32
7	63	21	16
3	9	3	16
42	×3×	1 ×	16=2016, Ans. gallons

PROB. VI.—*The sum of two numbers and their quotient given, to find those numbers.*

RULE.—Add 1 to the quotient and divide the sum of the two numbers by this sum, which will give the less number;

subtract the less number from the sum and you will have the greater number.

EXAMPLE.

Divide 100 into two such parts, that if the greater be divided by the less, the quotient may be just 30.

Thus, $30+1)100(3\frac{7}{31}$, less part; then $100-3\frac{7}{31}=96\frac{24}{31}$, greater number.

Prob. VII.—*The difference of two numbers and the quotient given, to find those numbers.*

Rule.—Divide the difference of the two numbers by the quotient, less 1, and you will have the less number. Add the less number to the difference, and this sum is the greater number.

EXAMPLES.

1. A grey hound, in pursuit of a hare, run three times as fast as the hare; and when he overtook her, he had run 30 rods more than she.

How many rods did each run?

Thus, $30\div\overline{3-1}=15$ rods, the hare run; then $15+30=45$ rods, the hound run.

2. A and B start at opposite points, to skate to the other's starting point: distance, 8 miles. A, by having the advantage (hence B, the disadvantage) of a uniform wind, performs his task $2\frac{1}{2}$ times the quickest, and 48 minutes the soonest. Required, the time that each is skating, and the force of the wind per minute.

Thus, $48\div\overline{2\frac{1}{2}-1}=32$ minutes, A's time; then $32+48=1$ hour and 20 minutes, B's time.

Then, $\overline{80+32}\div2=56$ minutes, each, if it had been calm; and $56-32=24$ minutes A was forwarded; and B was retarded the same.

Then, if $56:8::24:18102\frac{6}{7}$ ft., $\div24$ m. $=754\frac{2}{7}$ ft. per minute.

Prob. VIII.—*To multiply £ s. d. and qr. by £ s. d. and qr.*

Rule.—£×£=£; £ × s. = s.; £×d.=d.; £×qr.=qr.; s.×s.=20th of a s.; s.×d.=20ths of a d.; s.×qr.=20ths of a qr; d.×d.=240ths of a d.; d.×qr.=240ths of a qr.; and farthings multiplied by farthings, are 960ths of a farthing.

EXAMPLE.

Multiply £19 19s. 11d. 3 qr. by £19 19s. 11d. 3 qr.

OPERATION.

	£	s.	d.	qr.		£	s.	d.	qr.
Thus,	19	19	11	3					
	19	19	11	3					
	361	361	209	57	=	19	19	11	3×19£
	$\frac{361}{20}$	$\frac{361}{20}$	$\frac{209}{20}$	$\frac{57}{20}$	=	"	"	"	"×19s.
	£	s.	d.	qr.					

$$\left.\begin{array}{l} \text{£ } 19\times 11\text{d.}=209\text{d.} \\ \text{s. } 19\times \text{ " } =\frac{209}{20} \text{ of a d.} \\ \text{d. } 11\times \text{ " } =\frac{121}{240} \text{ of a d.} \\ \text{qr. } 3\times \text{ " } =\frac{33}{240} \text{ of a qr.} \end{array}\right\} = \begin{array}{cccc} \text{£} & \text{s.} & \text{d.} & \text{qr.} \\ 19 & 19 & 11 & 3\times 11\text{d.} \end{array}$$

$$\left.\begin{array}{l} \text{£ } 19\times 3\text{qr.}=57 \text{ qr.} \\ \text{s. } 19\times \text{ " } =\frac{57}{20} \text{ of a qr.} \\ \text{d. } 11\times \text{ " } =\frac{33}{240} \text{ of a qr.} \\ \text{qr. } 3\times \text{ " } =\frac{9}{960} \text{ of a qr.} \end{array}\right\} = \begin{array}{cccc} \text{£} & \text{s.} & \text{d.} & \text{qr.} \\ 19 & 19 & 11 & 3\times 3\text{qr.} \end{array}$$

£	s.	d.	qr.	
19	0	11	$3\frac{641}{960}$,	sum of the fractions.
380	18	2	2,	sum of the whole numbers.
399	19	2	$1\frac{641}{960}$,	Ans.

Hence, the product of two denominations, having the same integer, takes the name of the least, and is of such value as the larger denomination implies. Thus, 6s.×7d.=$\frac{42}{20}$d.; here the product takes the name of pence, and is 20ths, because the larger denomination is shillings, 20s. being the integer. Also, 6 inches×8 inches=$\frac{48}{144}$ of a foot, or=4 inches, *etc.*

PROB. IX.—*To divide a larger denomination by a smaller denomination, when the divisor and dividend used in such a division have the same integer.*

RULE.—Reduce the number to be divided to the same denomination as the divisor. Divide, and the quotient is the answer, in the same denomination as was the dividend, before it was reduced to the denomination of the divisor.

EXAMPLES.

1. Divide \$1 by 4 cts.; or 1 by ,04 :—Thus, \$1=100 cts., then 100÷4=\$25, Ans.; or 1,00÷ ,04=25, Ans.

2. Divide £1 by 1s.—First, £1=20s., then 20÷1= £20, Ans.

3. Divide 3 bushels by 3 quarts.—First, 3 bush.=96 qts., then $96 \div 3 = 32$ bushels, Ans.

4. Divide 4 acres by 5 rods.—First, 4 acres=640 rods; then $640 \div 5 = 128$ acres, Ans.

NOTE 1.—The preceding examples exist only in theory.

2. The two foregoing problems and their examples, involve the principles of cross multiplication and the reverse.

3. See the operation of the thirty-fourth question in Proportionals.

TO ABBREVIATE THE OPERATIONS OF ARITHMETIC.

PROB. I.—*To abbreviate operations in Multiplication and Division.*

RULE. 1.—Draw a perpendicular line, and place the numbers to be multiplied on the right, and the numbers by which you are to divide, on the left hand.

2. If there be two equal numbers on each side of the line, cross them out, and omit them in the operation.

3. If a number on one side of the line will divide a number on the other side, without a remainder, erase both numbers, and substitute for the larger the number of times it contains the smaller. Multiply the remainders together, on the right, for a dividend, and the remainders on the left, for a divisor.

EXAMPLE.—Multiply 8 by 9, and divide by 8; multiply the quotient by 6, and divide the product by 3.

Operation.

	$\dot{8}$	
$\dot{8}$	9	
$\dot{3}$	$\dot{6}$	2
	18, Ans.	

N.B.—For want of proper type, a dot is placed atop the figures to be erased.

PROB. II.—*To abbreviate the operation of the Multiplication and Division of Fractions by whole numbers—whole numbers by Fractions, or Fractions by Fractions.*

RULE. Draw a perpendicular line and place all those figures, which are to be multiplied together for a numerator, or dividend, on the right of the line, and those figures which are to be multiplied together for a denominator, or divisor, on the left hand of the line; also, the numerators of fractions, by which a division is to be made, on the left.

The question thus stated, equals on each side of the line may be crossed out.

When no two numbers remain, one on each side of the line, capable of being divided by any one figure, (see preceding operation,) multiply the figures on the right of the line for a numerator, or dividend, and those on the left for a denominator, or divisor, and the result will be the answer in the lowest terms of the fraction.

PROB. III.—*To abbreviate the operation of all proportional questions.*

RULE. 1.—Draw a perpendicular line, and place the sign of the answer on the left, at the top of the line, and that number which is of the same kind with the answer on the right, at the bottom; and as this number is greater or less than the answer sought, place the greater or less of the two remaining numbers on the right, and the other on the left, and proceed in all respects as before instructed, if Direct or Inverse; but, if Double Proportion, place any two of the same kind, of the remaining numbers, one on the right and the other on the left, according to directions for Direct or Inverse Proportion.

2.—Then cross out, multiply, and divide, as before directed.

NOTE 1.—When the answer is required in a different denomination from that given in the supposition, follow the tables from the denomination given, to the denomination required.

NOTE 2.—Mixed numbers must be reduced to improper fractions, and the numerators placed on that side of the line where the whole numbers, standing in the place of the fraction, would be placed.

EXAMPLE.

1. If 1 pint cost 10d. what will 3 hhds. cost in pounds?

Operation. *Explanation.*

How many £?	3 hhds.	Reducing hhds. to pints, see Red. Des.
hhd. 1	63 gal.	
gal. 1	4 qts.	
qt. 1	2 pts.	
pt. 1	10d.	Reducing *d.* to £, see Red. Ascen.
d. 12	1s.	
s. 20	1£	

Here, the answer is required in a different denomination than that given in the supposition. (See Note 1.)

Therefore, follow the tables, until you find the name of the answer required, observing to commence each successive step on the left with the denomination last placed on the right.

This method of operation renders the process of Reduction Descending and Ascending, entirely unnecessary.

2. A merchant bartered $5\frac{5}{8}$ cwt. of sugar, at $6\frac{3}{4}$d. per lb., for tea, at $8\frac{5}{8}$s. per pound. How much tea did he receive?

To discover to the student the utility of the abbreviating method, in operations, and the superiority it has over all other modes of performing such solutions as may be performed by the abbreviating process, let the preceding question be solved by the two different operations.

FRACTIONAL OPERATION OF BOTH METHODS.

$5\frac{5}{8}=\frac{53}{8}$ cwt., $\frac{53}{8}$ of $\frac{112}{1}=\frac{5936}{8}$	$5\frac{5}{8}=\frac{53}{8}$ cwt. For this part of *this*
$6\frac{3}{4}=\frac{27}{4}$ d., $\frac{27}{4}\times\frac{5936}{8}=\frac{160272}{36}$	$6\frac{3}{4}=\frac{27}{4}$ d. Method of opera-
$8\frac{5}{8}=\frac{69}{8}$ s., $\frac{69}{8}$ of $\frac{12}{1}$d. $=\frac{828}{8}$	$8\frac{5}{8}=\frac{69}{8}$ s. tion, see Note 2d.

OPERATION BY THE COMMON METHOD—FINISHED.

Thus, as $\frac{8}{828} : \frac{160272}{36} : : \frac{1}{1}$

```
   36
 ─────
 4968  160272
2484        8
──────────────────
29808)1282176(43 1/69 lbs., Answer.
       119232
       ──────
        89856
        89424
        ─────
```

Remainder, $\frac{432}{29808}=\frac{36}{2484}=\frac{3}{207}=\frac{1}{69}$.

OPERATION BY ABBREVIATING—FINISHED.

Tea,	53, sugar.
$\dot{9}$	
cwt. 1	$\dot{1}\dot{1}\dot{2}$ lbs. $\times 7\times 8\times 53=2968$
lb. 1	27d. $\dot{3}$
$\dot{4}$	
$\dot{4}$d. $\dot{1}\dot{2}$	1s.
s. 69	8
	1 lb. tea.
69	$2968=43\frac{1}{69}$ lbs., Answer.

3. If 4 men can build a wall in 20 days, in how many days could 8 men build the same wall?

Operation.

In how many days?	4 men.
2 men. 8	20d. 10
	10 days, Ans.

4. If 3 men can build 360 rods of wall in 24 days, how many rods can 8 men build in 27 days?

Note. 1. The student will see but 15 figures in the operation of this example; but, by the common method, it requires forty-four.

Also, see Example 2.

Operation.

How many rods?	8
3	27 3
3 24	360
	Rods, 1080, Ans.

Note 2. The ingenious pupil will easily discover to which *proportion* each of the preceding questions belong.

Remark.—Persons intending to teach, should become thoroughly acquainted with the three preceding problems, if they wish to enjoy the desirable advantage of *but one mode of solution, to the questions of several different rules.*

Considerable should be said by committees and teachers, in favor of pupils being taught this advantageous method of performing solutions.

Abbreviating operations is an amusement; and the necessary combination of numbers, to close a question by this process, is the *simplest* of the different methods, and when once practised, it can never be supplanted from the memory.

The scholar will find the most difficult questions to yield readily to this mode of solution, and has the satisfaction of proceeding upon a principle which is evidently unerring.

It is applicable to all Proportional Questions, embracing the Rules of Three, Single and Double, Direct and Inverse; Interest, Discount, Barter, Loss and Gain, Exchange, Reduction, Multiplication and Division of Fractions, &c. &c.

X. DECIMAL FRACTIONS.*

The reason of what is most difficult to understand in Decimal Fractions, is that of Multiplication, Division, and Reduction.

* *Decimal* is derived from the *Latin* word *decem*, which signifies *ten*. *Fraction* is derived from the *Latin* word *frango*, which signifies *to break*.

MULTIPLICATION AND DIVISION.

DEMONSTRATION.

1. It is obvious, that multiplying whole numbers by any fraction, is taking a certain part of the multiplicand for the product; consequently, multiplying one fraction by another, must produce a fraction smaller than either of the factors. And it is evident, that ,25 is $\frac{25}{100}$ or $\frac{1}{4}$, and ,5 is $\frac{5}{10}$ or $\frac{1}{2}$; and it is obvious, that $\frac{1}{2}$ of $\frac{1}{4}$ of $\frac{1}{1}$, is $\frac{1}{8}$ of $\frac{125}{1000} \times \frac{8}{1}$, which is ,125, because decimals read the same as whole numbers, and are the same as their equivalent vulgar fractions; therefore, $,25 \times ,5 = ,125$; consequently, the number of decimal places in any product, must be equal to the number of decimal places in both the factors of that product: *Hence the* RULE.

2. The preceding shews, that the product must have as many decimal places as both its factors.

The Multiplicand and Multiplier, in proving Multiplication, becomes the divisor and quotient in Division. Therefore, the number of decimal places in the quotient, must be equal to the difference between the number of decimal places in the dividend, and the number of decimal places in the divisor: *Hence the* RULE.

REDUCTION.

DEMONSTRATIONS.

1. *To Reduce a Vulgar Fraction to its equivalent Decimal.*

Let the given fraction, whose decimal expression is required, be $\frac{9}{15}$.

Now, since every decimal fraction has 10, 100, 1000, &c. for its denominator: and, if two fractions be equal, it will be, as the denominator of one is to its numerator, so is the denominator of the other to its numerator.

Thus, as $15 : 9 :: 10 : 6, = \frac{6}{10}$, the numerator required, and is the same as by the rule.

2. *To Reduce Numbers of different denominations to their equivalent decimal values,—and the contrary.*

This is only expanding or contracting the ratios in question, as they are larger or less than the ratio required; and what is taken from one ratio, to make it equal with the tenfold one, is only giving to another place what would have been left in its preceding place, had the *common* ratio been equal to the one in question.

And each of the given denominations, is but parts of the whole value, and may be treated of separately, which would make each denomination "*a vulgar fraction to be reduced to its equivalent decimal.*"

For an example, reduce 17s. 8¾d. to the decimal of a pound. Thus, ¾ of a penny=,75; consequently, 8¾d.=8,75d., but 8,75 is $\frac{875}{100}$ of a penny, or $\frac{875}{1200}$ of a shilling, which reduced to a decimal, is ,729166 &c. s. In like manner, 17,729166 &c. s. are $\frac{17729166}{20000000}$=,886458 &c., as by the rule.

From the preceding, we may infer, that the denominator of a higher denomination, is the divisor of its lower denomination, in reducing it to its equivalent decimal. Also, it is the multiplier for taking its numerator from the decimal.

On this and the preceding article, is founded the rules to the two last propositions.

3. *To find the decimal of any number of shillings, pence, and farthings, by inspection,—and the contrary.*

The invention of the rule is as follows:—As shillings are so many 20ths of a pound, half of them must be so many 10ths, and, consequently, take the place of 10ths in the decimal; but when they are odd, their half will always consist of two places, the *first* will be half the greatest even number, and the *second* the odd shilling, which is ,05 of a pound.

Again, farthings are so many 960ths of a pound; and had it happened that 1000 had been the pound, it is plain any number of farthings would have been so many thousandths, and might have taken their place in the decimal accordingly.

But 960, increased by a 24th part of itself, is 1000; consequently, any number of farthings, increased by their 24th part, will be an exact decimal expression: but, if the number of farthings be more than 12, a 24th part is more than ½ a farthing; therefore, 1 may be added; and when the number is more than 36, the 24th part is more than 1½ *qr.*, and therefore, 2 is added: *Hence the* RULES.

XI. INFINITE DECIMALS.

1. Decimals, consisting of figures continually repeated, are called Infinite Decimals, or Repetends; and arise from Vulgar Fractions, whose denominators do not measure their numerators.

The repetition of one figure, is a Single Repetend.

The repetition of one or more figures, is called a Compound Repetend. Thus, .33 is a single repetend; .3232 is a compound repetend.

When other figures occur before the repeating decimals, it is a Mixed Repetend—thus, .21321.

Single and compound circulates may be mixed repetends.

A single repetend (33) is expressed thus, $.\dot{3}$; a compound repetend (2121) thus, $.\dot{2}\dot{1}$.

Similar repetends—whether single or compound—are those which begin at the same place, either before or after the separatrix.

Dissimilar repetends are those which begin at different places.

Conterminous repetends are those which end at the same place.

Similar and Conterminous repetends are those which begin and end at the same place.

The denominator of the common fraction $\frac{1}{9}$, does not measure its numerator; hence $\frac{1}{9}$ reduced to decimals, is $.\dot{1}$, and is the equivalent decimal to $\frac{1}{9}$, then $.\dot{2}=\frac{2}{9}$, and so on, till $.\dot{9}=\frac{9}{9}$.

Therefore, every single repetend is equal to a vulgar fraction, whose numerator is the repeating figure, and denominator 9.

Again, $\frac{1}{99}=.0\dot{1}$; and $\frac{2}{99}=.0\dot{2}$, and the same holds true *ad infinitum*.

A mixed repetend consists of two parts—the finite part, and the repeating part.

The finite part may be reduced to its equivalent decimal, by "Reduction" of Decimals, *Article first*, in which it is demonstrated, and the infinite part may be treated as a single, or compound repetend, just as the case may be.

Hence the mixed repetend $(.15\dot{3})$ is reduced to its equivalent decimal—thus, Den. of the infinite part is 9,+00(the 0s for the 2 finite places)=900, then the 9,×15 (finite part) ,+3 (infinite part)=.138 Num.; therefore, $.15\dot{3}=\frac{138}{900}=\frac{23}{150}$, and the same of any mixed repetend.

Any given repetend, whether single, compound, pure, or mixed, may be transformed into another repetend, which shall consist of an equal or greater number of figures, at pleasure—thus, $.\dot{3}$ may be transformed into $.3\dot{3}$, or $.33\dot{3}$, &c.

3. *Similar and Conterminous* repetends should begin and end just as far from unity, as the farthest among the dissimilar

repetends, in addition of infinite decimals; and the quotient of the repetends, divided by as many 9s as there are places in the repetend, should be added to the finite part; and the remainder is the repetend of their sum.

4. If the repetend of a number to be subtracted, in subtracting infinite decimals, be greater than the repetend of the number it is to be taken from, then the right hand of the remainder must be less by unity than it would be if the expressions were finite.

5. *To multiply or divide Infinite Decimals*, change them to common fractions, multiply or divide, turn the fraction expressing the quotient or product, into its equivalent decimal, and you have the fraction required.

6. The following hints may serve to find whether the decimal fraction, equal to a given vulgar one, is finite or infinite, and of how many places the repetend will consist.

In dividing 1.000, &c., by any prime number, except 2 or 5, the figures in the quotient will begin to repeat over again, as soon as the remainder is 1.

And since $.99\dot{9}$ is less than 1, therefore, $.99\dot{9}$ divided by any number, will, when the repeating figures are at their period, leave 0 for a remainder.

The number of repeating figures we have, when the dividend is 1, will be the same number when the dividend is any other number.

Every repetend equally multiplied, must give the same product: for, if the products have more places, the surplus in each being alike, will be carried to the next, by which means, each product will be equally increased; consequently, every extended repetend will continue alike; and hence it appears that any *dividend* may be altered at pleasure, and the number of places in the repetend will still be the same.

XII. VULGAR FRACTIONS.

DEMONSTRATIONS.

The reason of what is most difficult to understand in vulgar fractions, is the rules for finding the greatest common measure, and the least common multiple; any of the remaining rules, with a little consideration, will appear very obvious.

PROBLEMS IV. and V. suggest to find the greatest common

measure and the least common multiple of two or more numbers.

1. What is the greatest common measure of 918, and 1998.

OPERATION.

First, 918)1998(2	Second, 162)918(5
1836	810
Rem. 162	108
Third, 108)162(1	Lastly, 54)108(2
108	108
54	

EXPLANATION.

The truth of the rule will be discovered by retracing the preceding operation, as follows:

Since 54 (the last divisor) measures 108, it also measures $108+54$, or 162. Again, since 54 measures 108 and 162, it also measures $5\times162+108$, or 918. In the same manner it will be found to measure $2\times918+162$, or 1998. Therefore, 54 measures both 918 and 1998, and is their *greatest* common measure; for, suppose there be a greater—then, since the greater measures 918 and 1998, it also measures the remainder, 162; and since it measures 162 and 918 it also measures the remainder 108; in the same manner it will be found to measure the remainder, 54; that is, the greater measures the less, which is absurd.

2. *Remarks respecting the least common multiple of two or more numbers.*

It is obvious that one number is the multiple of another, when the former contains all the factors of the latter.

The factors of 6 are 3 and 2, and the factors of 9 are 3 and 3. Now 54 contains all these factors, $(3\times2\times3\times3=54)$, and 54 is a common multiple of 6 and 9, but it is not their *least* common multiple—it is 3 times as great as the least, owing to the existence of the factor, 3, in both 6 and 9. Hence we observe, that a *common factor* of two or more numbers must enter but once into the multiplication, to give the least common multiple.

The necessary exclusion is effected by the following:

RULE.—*Divide two or more of the given numbers by any prime number that will measure them; repeat the operation upon the quotients and undivided numbers, and thus continue, till they*

become prime to each other. Multiply the several divisors, the last quotients, and undivided numbers together; the product will be the least common multiple.

3. *To change two or more fractions which have different denominators, to equivalent fractions, having the same denominator.*

It is evident, that if both the numerator and denominator of a fraction are increased or diminished alike, the fraction remains of the same value; and as many times the numerator is made greater, or smaller, so many times the fraction is made greater or smaller, and as many times as the denominator is made greater so many times the fraction is made smaller, or, as many times as the denominator is made smaller, so many times the fraction is made greater: Hence the reason of the *rule*, and hence the rules for the multiplication and division of fractions.

The reason of the divisor's being inverted, in division of fractions, is because it is supposed to be written under the dividend and then removed by multiplying the numerator of the dividend by the denominator of the divisor, and the denominator of the dividend by the numerator of the divisor.

EXAMPLE.—Divide $\frac{3}{4}$ by $\frac{3}{8}$.

Operation by the rule; thus, $\frac{8}{3}\times\frac{3}{4}=\frac{24}{12}=2$, Ans.

Demonstration of the rule; thus, $\left\{ \frac{\frac{3}{4}}{\frac{3}{8}}=\frac{\frac{24}{4}}{3}=2\right.$, Ans.

4. To reduce a fraction of a lower denomination to the fraction of a higher; or, reducing fractions of a higher, to fractions of a lower denomination, is only reducing them to compound fractions, by comparing the *given* fraction with all the denominations between itself and the denomination to which you would reduce it.

EXAMPLE.—Reduce $\frac{3}{10}$ of a mill to the fraction of a dollar.

By comparing it, it becomes a compound fraction, thus:—
$\frac{3}{10}$ of $\frac{1}{10}$ of $\frac{1}{10}$ of $\frac{1}{10}=\frac{3}{10000}$, Ans.

The reverse, thus: $\frac{3}{10000}$ of $\frac{10}{1}$ of $\frac{10}{1}$ of $\frac{10}{1}=\frac{3}{10}|\frac{000}{000}$, Ans.

5. Fractions, before they are reduced to a common denominator, are entirely dissimilar, and therefore cannot be incorporated with one another; but when they are reduced to a common denominator, &c. and made parts of the same thing; their sum, or difference, may then be as properly expressed by the sum or the difference of the numerators, as the sum or difference of any two quantities, by the sum or difference of their individuals; hence the reason of the rules, for addition and subtraction of fractions.

XIII. PROPORTION.

EXPLANATIONS.

1. The idea, annexed to the term, *proportion*, is easily conceived.

The rule itself is founded on this obvious principle. "The magnitude or quantity of any effect varies constantly in proportion to the varying part of the cause."

Proportion is distinguished into continual and discontinual.

If, of several couplets of proportionals, written down in a series, the difference or ratio of each consequent, and the antecedent of the next following couplet, be the same as the common difference or ratio of the couplets, the proportion is said to be continual, and the numbers themselves, a series of *continual proportionals, either arithmetical or geometrical.*

2. Four numbers are said to be in *direct* proportion, when *more* requires *more*, or *less* requires *less*.

More requires *more* when the third term is greater than the first, and requires the fourth term to be greater than the second.

Less requires *less*, when the third term is less than the first, and requires the fourth term to be less than the second.

3. Four numbers are said to be *reciprocally* or *inversely* proportional, when the fourth is less than the second, by as many times as the third is greater than the first, or when the first is to the third, as the fourth to the second, and *vice versa.** Hence the phrase—if *more* requires *less* or *less* requires *more*, the proportion is *Inverse.*

4. *Harmonical Proportion* is that, which is between those numbers which assign the lengths of musical intervals, or the lengths of strings sounding musical notes; and of three numbers it is, when the first is to the third, as the difference between the first and second is to the difference between the second and third, as the numbers, 3, 4, 6.

Thus, if the length of strings be as these numbers, the sound will be an octave 3 to 6, a fifth 2 to 3, and a fourth 3 to 4.

Again, between four numbers, when the first is to the fourth, as the difference between the first and second is to the difference between the third and fourth, as in the numbers 5, 6, 8, 10: For the strings of such lengths will sound an octave 5 to 10; a sixth greater 6 to 10; a third greater 8 to 10; a third less 5 to 6; a sixth less 5 to 8; and fourth 6 to 8.

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* *Vice versa* is a Latin word signifying, *the terms being exchanged.*

Lastly—A series of numbers in harmonical proportion is, reciprocally, as another series in arithmetical proportion.

XIV. ALLIGATION.

DEMONSTRATION.

1. There is nothing in the different cases and operations in Alligation, the reason of which does not appear plain, except what relates to finding the quantities, at several different prices, to be mixed together in such proportion that one pound, bushel, &c. of the mixture, may be of a certain value, less than the *highest* and greater than the *lowest* price. The rule for doing this is called *Alligation Alternate*, from *alligo* to bind or connect together, and *alterno*, to change by turns; because the prices of different simples are linked together, and their differences are made to change places with one another.

The operation by this rule gives the true answer by connecting a greater and less, than the mean price, and placing their differences alternately for the quantities themselves, by which there is precisely as much gained by one quantity, as is lost by the other.

We may variously alligate the values of the ingredients, and thus obtain various results, all of which will be correct; and the results will be correct answers.

DEMONSTRATION.—Let two different qualities of grain be mixed together, one kind worth \$2 a bushel, and the other \$4, in such proportion that a bushel of the mixture shall be worth \$3.

Operation.

$$3\begin{cases} 2 — 1 \\ 4 — 1 \end{cases}$$

EXPLANATION.—Here it is found by the rule that there must be 1 bushel of each sort, and the price of both bushels, one at \$2 and the other at \$4, is \$6, which divided by 2, the number of bushels mixed, gives \$3, the price of a bushel of the mixture. It is also plain that the bushel which was worth \$4 before it was mixed, has lost \$1 by the mixture, and the bushel that was worth \$2 has gained \$1; therefore, there is as much gained by one, as is lost by the other, *which was to be proved.* The same principle will hold true in all cases, let the number of simples be what it may.

2. The difference between the greater and the less of two prices connected, is the common denominator of fractions, of which the differences between each extreme and the mean

price, are the numerators, which fractions always express the proportional quantities of the different things to be mixed, and by removing the common denominator, the numerators become whole numbers: hence the reason of the differences of the mean and extreme prices being the same as the number of things required to be mixed.

DEMONSTRATION.

Example.

$$5\left\{\begin{array}{l} 3-|\frac{3}{5} \text{ or } 3 \\ 8-|\frac{2}{5} \text{ or } 2 \end{array}\right.$$

Here is an example in which the mean price is 5, and the extremes are 3 and 8; the quantities to be mixed are 3 of the lesser price, and 2 of the greater. The difference between 3 and 8 is 5, which is the common denominator, and $5-3=2$, the numerator of the least fraction, against the greatest price, and $8-5=3$, the greatest numerator of the fraction or proportion at the least price. The proportions are now found in fractions; but it is plain that if the common denominator 5, be taken away from both, it is the same as multiplying the numerators by it, and then dividing the products by the denominator; therefore, 3 and 2 are whole numbers, and they are the least whole numbers that can have the same relation to each other as $\frac{3}{5}$ to $\frac{2}{5}$.

When several less numbers are connected with one greater, or the contrary, the principle is the same.

MISCELLANY.

1. Part 1500 acres of land between Saul, Seth and Silas; and give Seth 72 more than Saul, and Silas 112 more than Seth.

First, Seth's 72, +Silas' 72 and 112 more than Saul's, is 256 acres from 1500, leaving 1244 to be divided equally; hence, $1244\div3=414\frac{2}{3}$ acres, Saul's share.

Then, $414\frac{2}{3}+72$, Seth's; and $414\frac{2}{3}+72+112$, Silas' share.

2. What is the difference between six dozen dozen, and half a dozen dozen?

First, a dozen dozen is $144,\div2=72$, $\frac{1}{2}$ a dozen.

Then, $\underline{144\times6}-72=792$ difference, Ans.

3. What number, deducted from the 32d part of 3072, will leave the 96th part of the same?

First, $3072\div32=96$; then, $3072\div96=32$.

Then, $96-32=64$, Ans.

4. There is a certain number, which being divided by 7, the quotient resulting multiplied by 3, that product divided by 5, from the quotient 20 being subtracted; and 30 added to the remainder, the half sum shall make 35. What is the number?

Thus, $35 \times 2 - 30 + 20 \times 5 \times 7 \div 3 = 700$, Ans.

5. How many trees, four feet apart each way, may grow on an acre of ground?

First, 1 acre=43560 feet, and four feet every way=16 feet; then, $43560 \div 16 = 2722$ trees, Ans.

6. A sheepfold was robbed three nights successively; the first night, half the sheep were stolen, and half a sheep more; the second night half the remainder were stolen, and half a sheep more; the last night, they took half of what were left, and half a sheep more, by which time they were reduced to 30. How many were there at first?

Thus, $\frac{1}{2} + 30 = \frac{1}{2}$ of what were in the fold before any were taken the last night, therefore, $30{,}5 + 30{,}5 = 61$ in the fold before any were taken the last night, and half of them were stolen and half a sheep more, consequently, 31 taken the third night, 62 the second, and 124 the first.

Then, $30 + 31 + 62 + 124 = 247$, Ans.

7. What part of $33\frac{1}{21}$ is $28\frac{11}{12}$?

First, $33\frac{1}{21} = \frac{2776}{84}$, and $28\frac{11}{12} = \frac{2429}{84}$; then, $\frac{2429}{2776} = \frac{7}{8}$, Ans.

8. Find two numbers that $\frac{13}{16}$ of one shall equal $\frac{7}{16}$ of the other.

Thus, $\frac{16}{13} \times \frac{7}{16} = \frac{128}{208}$, Ans.

9. If $\frac{1}{3}$ of 6 be 3 what will $\frac{1}{4}$ of 40 be?

First, $\frac{1}{3}$ of 6 is 2, and $\frac{1}{4}$ of 40 is 10; then, if 2 is 3, 10 is 15: Ans. 15.

10. At what time, between twelve and one o'clock, do the hour and minute hands of a clock point in directions exactly opposite?

The minute hand must gain 30 minutes on the hour hand, before they will point in opposite directions, and the minute hand, in moving $1\frac{1}{11}$ minute, gains 1 minute, because the motion of the hands are in the ratio of 11 to 1; consequently, $30 \times 1\frac{1}{11} = 32\frac{8}{11}$ minutes past 12, Ans.

11. Seven-eighths of a certain number exceeds four-fifths of the same by 6; what is that number?

First, $40 \times 7, \div 8 = 35$, and $40 - 35 = 5$; $40 \times 4, \div 5 = 32$,

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and 40—32=8; and 8—5=3. Then if 3:6: :40:80, $\frac{7}{8}$ of which exceeds $\frac{4}{5}$ by 6.

12. From 14 years, take 11 yrs. 11 mo. 11 w. 11 d. 11 h. 11 m. 11 s.

	Y.	m.	d.	h.	m.	s.	
OPERATION.	14	00	00	00	00	00	
	12	1	4	11	11	11	
	1	11	23	12	48	49,	Ans.
Or,	1	11	3w. 2d.	"	"	"	

13. From 1 fur. take 39 rods, 4$\frac{2}{3}$ yds. 2 ft. 5 inches.

	Fur.	rd.	yd.	ft.	in.	
OPERATION.	1	00	00	00	00	
		39	4$\frac{2}{3}$	2	5	
	.	.	.	.	1,	Ans.

14. What is the gross weight of a hogshead of tobacco, weighing neat 11 cwt. 1 qr.; tare 14 lbs. per. cwt.?

First, 112—14=98; then, if 98:112: :11 cwt. qr.: 12 cwt. 3 qrs. 12 lbs., Ans.

15. The births, in a certain town, were 475, and the proportion, 13 boys to 12 girls; what was the number of each?

Thus, 13+12:475: :13:247 boys; then, 475—247=228 girls.

16. What sum of money will produce as much interest in 3$\frac{1}{4}$ years as $210,15 can produce in 5$\frac{5}{12}$ years?

First, 3$\frac{1}{4}$ yrs.=39 months, and 5$\frac{5}{12}$ yrs.=65 months.

Then, if 39:210, 15: :65:$350,25, Ans.

17. Divide the number 360 into four such parts, which shall be to each other as 3, 4, 5, and 6. First, 3+4+5+6=18.

Then, 18:360: :3:60 } Ans. And, 18:360: :5:100 } Ans.
" : " : :4:80 } " : " : :6:120 }

18. C hired A and B to cut wood; A could cut a cord in 4 hours, and C in 6 hours; in what time could both cut a cord?

*First, 6+4,÷2=5 of 12 hours; then, 5:60: :12:2 hours, 24 minutes, Ans.

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*The * signifies *the operation of the preceding question begins at that place.*

PROPORTIONALS.

I. This and the following division are difficult questions explained, a perusal of which will lead to their investigation, and consequently discipline the mind, giving clearness and activity of thought, strengthening the power of comprehension, and therefore tend to lay the foundation so indispensable to the formation of plain reasoning and a sound judgment.

II. Teachers may place the operations of the following questions upon the black-board, and define the reason of each step, to their pupils, should they be unable to give it.

1. A lion of bronze, placed at the brink of a reservoir, can spout water into it from his mouth and right foot. If he spouts from his mouth only, he will fill the reservoir in 50 minutes; if from his foot only, he will fill it in 40 minutes. It has a discharging faucet which will empty it in 25 minutes. In what time would the reservoir be filled, if permitted to spout from his mouth and foot; and in what time would it be filled if the faucet is left open, and he spouts from his mouth and foot?

*First, $\frac{1}{40}+\frac{1}{50}=\frac{9}{200}$ of it filled in 1 minute.

Then, if 9 : 1 : : 200 : $22\frac{2}{9}$ minutes, Ans. 1.

Then, what runs out in 1 minute, is equal to $\frac{8}{200}$ of what runs in, in 1 minute; therefore, in 1 minute $\frac{1}{200}$ of the reservoir is filled, when all are running; consequently, 200 minutes, 3 hours 20 minutes, Ans. 2.

2. A water-tub that holds 147 gallons, has a pipe that brings in 14 gallons in 9 minutes, and a *tap* that discharges 40 gallons in 31 minutes; now suppose these both to be left open by mistake at 2 o'clock, and a servant at 5, finding the water running, shuts the *tap*, and is solicitous to know in what time the tub will be filled after the discovery of the accident.

*First, from 2 to 5,=180 minutes; then, if 9 : 14 : : 180 : 280 gals. runs in, in 3 hours; then, if 31 : 40 : : 180 : $232\frac{8}{31}$ gals. runs out in 3 hours; and 280—$232\frac{8}{31}$=$47\frac{23}{31}$ gals. is in at 5 o'clock; then 147—$47\frac{23}{31}$=$99\frac{8}{31}$ gals. is to run in, after the discovery. Then, if 14 : 9 : : $99\frac{8}{31}$: 1 hour, 3 minutes and $48\frac{114}{217}$ seconds, Ans.

3. A certain schoolmaster was hired for one month upon these conditions, that if he had 20 scholars, he was to have $25; and if he had 30 scholars, he was to have $30. It so happened that he had 29; pray what was his wages?

*If 10 scholars less than 30 : add 25 cents to each of the 20 : : 9 scholars, more than 20, : diminishes $22\frac{1}{2}$ cts. from the 29 at \$1,25; then, $25-22\frac{1}{2}=2\frac{1}{2}$ cents per scholar more than one dollar each, consequently, \$29,72,5, Ans.

If 3 men, or 4 women, do a piece of work in 56 days, how long will one man and one woman be in doing the same?

*First, if $3:4::1:1\frac{1}{3}$, which plus $1=2\frac{1}{3}$.

Then, if $4:56::2\frac{1}{3}:96$ days, Ans.

5. A, B, and C bought a piece of land, the profits of which amount to £120 per annum. Now the sum of money that each paid, was in such proportion, that as often as A paid £5, B paid £7, and as often as B paid £4, C paid £6. How much must each have of the gain per annum?

* First, if 4 : 6 : : 7 : £10,5; then 10,5+5+7= £22,5. If 22,5 : 5 : : 120 : £$26\frac{30}{45}$, A's. As 22,5 : 7 : : 120 : £$37\frac{15}{45}$, B's. Again, 22,5 : 10,5 : : 120 : £56, C's part of the gain.

6. A person went to 4 taverns in succession, upon entering each of which, he borrowed as much money as he carried to it; and, upon leaving them, he paid the landlords one dollar each, which done, he finds himself without money. What sum of money did he carry to the first tavern?

* First, 100 cts., last landlord's bill, is composed of 50 cts. that he borrowed, and 50 cts, that he spent; for he carried to it and borrowed alike: consequently, if 100 : 50 : : 50 : 25 cts. spent at the *third* tavern. And if $50:25::25:12\frac{1}{2}$ cts.
" " *second* " and if $25:12\frac{1}{2}::12\frac{1}{2}:6\frac{1}{4}$ cts.
" " *first* " for the more he spent, the less money he would have, and consequently would borrow less. \$0.93.$7\frac{1}{2}$, Ans.

7. A can mow one acre of grass in $5\frac{2}{3}$ hours, and B can mow $1\frac{3}{4}$ acre in $9\frac{1}{3}$ hours. In what time can A and B, jointly, mow $8\frac{1}{4}$ acres?

* First, if $1\frac{3}{4}:9\frac{1}{3}::1:5\frac{1}{3}$ hours, B, in mowing one acre; then, $5\frac{1}{3}+5\frac{2}{3}=11$ hours=660 minutes, which gives 4 acres when both are mowing. Then, if $4:660::8\frac{1}{4}:1361\frac{1}{4}=22$ hours, 41 minutes, and 15 seconds, Ans.

8. If a Cardinal can pray a soul out of "purgatory," by himself, in one hour, a Bishop in 3 hours, a Priest in 5 hours, and a Friar in 7 hours. In what time can they pray out 3 souls, all praying together?

* First, $\frac{1}{1}+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}=\frac{176}{105}$. Then, if 176 : 105 : : 3 : 1 h. 47 m. $23\frac{2}{11}$ s., Ans.

9. A can do a piece of work in 10 days, B in 13 days. In what time could both together finish a piece of work?

* Thus, $\frac{1}{10}+\frac{1}{13}=\frac{23}{130}$, and 23 : 130 : : 1 : $5\frac{15}{23}$ days, Ans.

10. A can do a piece of work in 3 weeks, (six days to a week,) B can do thrice as much in eight weeks, and C 5 times as much in twelve weeks. If all work together, in what time can they finish a piece of work?

* First, $\frac{1}{3}+\frac{3}{8}+\frac{5}{12}=\frac{27}{24}$. Then, if 27 : 6 : : 24 : 5 days 4 hours, Ans.

11. A and B can do a piece of work in 5 days; A can do it alone in 7 days. In what time can B do it by himself?

* Thus, 7—5=2. Then, if 2 : 5 : : 7 : $17\frac{1}{2}$ days, Ans.

12. A can reap a piece of wheat in 5 days, B can reap it in 8 days, and A and C together in 4 days. How long would all three be in reaping it?

* First, $\frac{1}{5}$, $\frac{1}{8}$, $\frac{1}{4}=\frac{23}{40}$, which is made up of 2 days work by A, and 1 by B and C, each. Then $\frac{23}{40}-\frac{1}{5}$ (A's extra labor) $=\frac{15}{40}$, what all did. Then, if 15 : 40 : : 1 : $2\frac{2}{3}$ days, Ans.

13. A and B can build a boat in 18 days, and with the assistance of C, they can do it in 11 days. In what time can C do it?

* First, A and B, in one day, would do $\frac{1}{18}$ of the work, and A, B, and C would do $\frac{1}{11}$ of the work. Then, A, B, and C ,$\frac{1}{11}-\frac{1}{18}$ A+B$=\frac{7}{198}$, what C would do in 1 day. Then, if 7 : 198 : : 1 : $28\frac{2}{7}$ days, Ans.

14. A, B, and C agree to do a piece of work for £2 10s. A and B perform $\frac{3}{11}$ of the work, A and C $\frac{5}{13}$, B and C $\frac{4}{14}$ of it. How much does each receive, if paid proportionably?

* First, A+B$=\frac{3}{11}$=,2727; A+C$=\frac{5}{13}$=,3846; and B+C$=\frac{4}{14}$=,2857; then, ,2727+,3846+,2857=,943, twice as much as they all performed; therefore, ,943÷2=,4715=A, B, and C. Then, ,4715—,2857=B+C=,1858=A; and ,4715—,3846=,0869=B; and ,4715—,2727=,1988=C.

Then, as ,4715 : 50s. : : ,1858 : 19s. 8d. $1\frac{3445}{4715}$qr. A receives.
Also, as " : " : : ,0869 : 9s. 2d. $2\frac{1570}{4715}$qr. B "
And, as " : " : : ,1988 : £1 1s. 0d. $3\frac{300}{4715}$qr. C "
Ans.

15. A, B, and C are employed to do a piece of work for $26,45 cts.; A and B are supposed to do $\frac{3}{4}$ of the work; A and C $\frac{9}{10}$; B and C $\frac{13}{20}$; and are paid proportionably. What share has each of the money?

* First, $\frac{3}{4}+\frac{9}{10}+\frac{13}{20}=\frac{46}{20}$, but separately, they are $\frac{15}{20}$, $\frac{18}{20}$, $\frac{13}{20}$,

and $\frac{46}{20}$ is twice as much as they all performed; hence $\frac{46}{20} \div 2 = \frac{23}{20}$.

Then, $\frac{23}{20} = \overline{\text{A, B, and C}} - \frac{13}{20}$, $\overline{\text{B and C}} = \frac{10}{20}$, what A performed; also, $\frac{23}{20} - \frac{18}{20} = \frac{5}{20}$, what B did; and $\frac{23}{20} - \frac{15}{20} = \frac{8}{20}$, what C did.

The sum of the numerators of the parts of what each performed, is 23; hence, A's part is $\frac{10}{23}$, B's $\frac{5}{23}$, C's $\frac{8}{23}$.

Then, if 23 : 26,45 : : 10 : $11,50 cts., A's share.		
Also, if " : " : : 5 : 5,75 cts., B's "		Ans.
And, if " : " : : 8 : 9,20 cts., C's "		

16. A and B can dig a trench in 4 days, B and C in 6 days, A and C in 5 days. Required, the time that A, B, and C would do it, and the time that each would do it.

* Now, A and B, in one day, would dig $\frac{1}{4}$, B and C $\frac{1}{6}$, A and C $\frac{1}{5}$. The sum of these parts is $\frac{37}{60}$, and is what all would do in 2 days, because each is mentioned twice. Then $\frac{1}{2}$ of $\frac{37}{60} = \frac{37}{120}$, what all would do in 1 day; then if you subtract, separately, what each two would do in 1 day, from $\frac{37}{120}$, you will have what each one would do in 1 day, which is $\frac{17}{120}$ for A, $\frac{13}{120}$, for B, and $\frac{7}{120}$ for C. The sum of the numerators=37.

Then,
37 : 120 : : 1 : $3\frac{9}{37}$ days, all are in digging the trench
17 : " : : 1 : $7\frac{1}{17}$ " A is " "
13 : " : : 1 : $9\frac{3}{13}$ " B " " "
7 : " : : 1 : $17\frac{1}{7}$ " C " " "
Answer.

17. If 15 men can perform a piece of work in 11 days, how many men will accomplish another piece of work, four times as large, in a fifth part of the time?

* First, as 1 : 15 : : 4 : 60 men; then, as 1 : 60 : : 5 : 300 men, Ans.

18. A, B, and C can complete a stair-case in 12 days; A can make it alone in 23 days, and B in 37 days? In what time can C do it by himself?

* First, days, 12 : 1 : : 1 day : $\frac{1}{12}$ of it.
23 : 1 : : 1 " : $\frac{1}{23}$ "
37 : 1 : : 1 " : $\frac{1}{37}$ "

Then, $\frac{1}{12} - \frac{1}{23} + \frac{1}{37} = \frac{131}{10212}$ of it, C completes in one day.

And, 131 : 1 : : 10212 : $77\frac{125}{131}$ days, Answer.

19. A master mason is offered $25 for doing a piece of work, which he can finish in $12\frac{1}{2}$ days; and his journeyman can do it in $18\frac{3}{4}$ days, his apprentice in $22\frac{1}{2}$ days. If they work together, in what time would they finish it, and how much would each earn?

* First, all working together, would finish $\frac{14}{75}$ of it in 1 day.

Then, if $14 : 1 : : 75 : 5\frac{5}{14}$ days, all would be in doing it; and the master mason does $\frac{6}{13}$ of it, the journeyman $\frac{4}{13}$, and the apprentice $\frac{3}{13}$.

Then, if $13 : 25 : : 6 : \$11\frac{7}{13}$, the master mason earns.
And, if " : " : : $4 : \quad 7\frac{9}{13}$, " journeyman " Ans.
" " : " : : $3 : \quad 5\frac{10}{13}$, " apprentice "

20. A man bought apples at 5 cents a dozen, half of which he exchanged for pears, at the rate of 8 apples for 5 pears; he then sold all his apples and pears at a cent a piece, and thus gained 19 cts. How many apples did he buy, and how much did they cost?

* First, 1 apple$=\frac{5}{12}$ of a cent, and 8 apples$=\frac{40}{12}$ of a cent, and 5 pears$=\frac{40}{12}$; hence 8 apples and 5 pears$=\frac{80}{12}=\frac{20}{3}=6\frac{2}{3}$ cts.,$=\frac{20}{39}$ of a cent for 1, and 1 sold for $\frac{39}{39}$ of a cent, then, $\frac{39}{39}-\frac{20}{39}=\frac{19}{39}$ of a cent gained on 1.

Then, if $\frac{19}{39} : 1 : : 19 : 39$, then $8+5 : 39 : : 8 : 24=\frac{1}{2}$ of the apples; therefore, 48 apples in all: and, if 12 apples : 5 cts., : : 48 : 20 cts., they cost.

21. A bought a load of wood, measuring 2 cords, in sticks of 12 feet in length, and agreed with B to saw the whole to 2 feet lengths, at the usual price of 50 cents per cord, of 4 feet lengths. How much should B be paid for sawing it?

* First, he cut each stick into 4 feet lengths, hence twice in two; then each of these three sticks once in two again, hence each 12 ft. stick is sawed 5 times in two.

Then, if $2 : 50 : : 5, \times 2 : \$2,50$ cts., Ans.

22. A person being asked the time of day, said it was between 3 and 4 o'clock, and the hour and minute hands were together. Required, the exact time.

* Now, since the minute hand moves 12 spaces to the hour hand one, the minute hand moves 11 times the fastest; hence, *as* 11 *is to* 1, *so is* 12, *multiplied by the number of spaces that both hands are from* 12, *to the time they will be together again.* Consequently, as $11 : 1 : : 12, \times 3 : 3$h. $16\frac{4}{11}$m. (*i.e.* $16\frac{4}{11}$m. past 3 o'clock, Ans.)

Or, thus, as $11 : 3 : : 12 : 3$h. $16\frac{4}{11}$m., which is the same.

23. A gentleman being asked what o'clock it was, said that it was between 5 and 6; but, to be more particular, he said that the minute hand had passed as far beyond the 6, as the hour hand wanted of being to the 6; that is, the hour and minute hands made equal acute angles, with a line passing from the 12 through the 6. Required, the time of day.

* *Questions of this nature, are solved as follows:*

As 13 : 6 : : 12 : 5h. 32m. $18\frac{6}{13}$s. (*i.e.* 32m. $18\frac{6}{13}$s. past 5 o'clock, Ans.)

24. A person looking on his watch, was asked the time of day; he said the time past noon was equal to $\frac{4}{5}$ of the time to midnight. Required, the exact time.

* From noon to midnight is to be divided into two parts, in the ratio of 4 to 5. Hence, as 4+5 : 4 : : 12 : 5h. 20m., Ans.

25. Two persons, comparing their ages, find them, at present, in the ratio of 7 to 5, but, that 30 years ago, they were as two to one. What is the age of each?

* Inversely, 7+5 : 30 : 2+1 : 120, the sum of their ages.

Then, as 7+5 : 120 : : 7 : 70 yrs., the older.

And, 120—70=50 yrs., age of the younger.

26. A son having asked his father's age, the father thus replied: Your age is 12 years, to which, if $\frac{5}{8}$ of both our ages be added, the sum will express my age. What was the father's age?

* First, add $\frac{5}{8}$ of 12 to 12=$19\frac{1}{2}$, which wants $\frac{5}{8}$ of the father's age to complete the answer; hence $19\frac{1}{2}$=$\frac{3}{8}$ of the father's age.

Then, if 3 : $19\frac{1}{2}$: : 8 : 52 yrs., the father's age, Ans.

27. A's age is double that of B, and B's triple that of C, and the sum of all their ages is 140. What is the age of each?

* First, call C's age 1 part of the 140 years; then B's would be 3 parts, and A's 6 parts; then, 1+3+6=10 parts in the sum of their ages.

$$\text{Then, if}\left\{\begin{array}{l}10 : 140 : : 1 : 14 \text{ yrs., age of the youngest.}\\ \text{“} : \text{“} : : 3 : 42 \text{ yrs., B's age.}\\ \text{“} : \text{“} : : 6 : 84 \text{ yrs., A's “}\end{array}\right\}\text{Ans.}$$

28. A says to B, two-thirds of my age is equal to three-fourths of yours, and their difference is 10 years. What is the age of each?

* First, $\frac{2}{3}$, and $\frac{3}{4}$, reduced to a common denominator,=$\frac{8}{12}$, and $\frac{9}{12}$; therefore, their ages are in the ratio of 8 to 9, and 10 years must be equal to $\frac{1}{9}$ of A's age, and $\frac{1}{8}$ of B's.

$$\text{Then, if}\left\{\begin{array}{l}\frac{1}{9} : 10 : : \frac{9}{9} : 90 \text{ yrs., A's age.}\\ \frac{1}{8} : \text{“} : : \frac{8}{8} : 80 \text{ yrs., B's “}\end{array}\right\}\text{Ans.}$$

29. A man had four sons, whose ages differed from each other 4 years, and the youngest was half as old as the oldest. Required, the age of each.

* First, call 4 years a part; then, the 3d son is 1 part, the 2d 2, and the first or oldest three parts the most; hence 3 parts is the age of the youngest, and 6 parts is the age of the oldest, for one is half as old as the other.

Consequently, by proportion, their ages, separately, are 12, 16, 20, and 24 yrs., Ans.

30. B's age is $1\frac{1}{2}$ the age of A, and C's $2\frac{1}{10}$ the age of both; and the sum of their ages is 93. What is the age of each?

* First, 8, A's age.
Then, 12, B's "
And 42, C's "

Hence, 62, their sum.

Then, if $\begin{cases} 62 : 93 : : \ 8 : 12 \text{ yrs., A's age.} \\ \text{"} : \text{"} : : 12 : 18 \text{ yrs., B's "} \end{cases}$

Consequently, $\overline{30} \times 2\frac{1}{10} = 63$ yrs., C's age.

31. A hare is 50 of its own leaps before a pointer, and takes 4 leaps to the pointer's 3; but two of the pointer's leaps are equal to 3 of the hare's. How many leaps must the pointer take to catch the hare?

* First, as $3 : 2 : : 4 : 2\frac{2}{3}$; then, $3 - 2\frac{2}{3} = \frac{1}{3}$ of a leap gained every 2 leaps: consequently, $\frac{1}{6}$ of a leap at 1 leap; hence, 50 leaps are gained in 300 leaps, Ans.

32. A hare starts 12 rods before a hound, but is not perceived by him until she has been up 45 seconds; she scuds away at the rate of 10 miles an hour, and the hound, on view, makes after her at the rate of 16 miles an hour.

In what time will he catch her, and how far will he run?

* First, 1 hour=3600s., and 10 miles=3200 rods; then, if she runs in 3600s. : 3200 : : 45s. : 40 rods, plus 12=52 rods, the hare gets before the hound starts; then 16—10=6=1920 rods gained in 3600s.

Then, if $\begin{cases} 1920 : 3600 : : 52 : 97\frac{1}{2}\text{s. in catching the hare.} \\ 3600 : 16 \text{ miles} : : 97\frac{1}{2} : 2288\text{ft. the hound runs.} \end{cases}$ Ans.

33. A, B, and C agree to contribute \$730 towards the building of a church, at the distance of 2 miles from A, $2\frac{7}{8}$ miles from B, and $3\frac{1}{2}$ miles from C; and they agree, that their shares shall be reciprocally proportional to their distances from the church. Required, what each contributes.

* First, $2 \times 8 = \frac{16}{8}$; then $2 \times 8, +7 = \frac{23}{8}$, and $3\frac{1}{2} = \frac{28}{8}$; therefore, the first pays \$28 as often as the second \$23, and the third \$16; and C=16,+B=23,+A=28,=67.

Then, if $\begin{cases} 67 : 730 : : 16 : \$174,32\frac{56}{67}, \text{ C must pay.} \\ \text{"} : \text{"} : : 23 : \$250,59\frac{47}{67}, \text{ B } \quad \text{"} \\ \text{"} : \text{"} : : 28 : \$305,07\frac{31}{67}, \text{ A } \quad \text{"} \end{cases}$ Ans.

Proof, \$730.

34. The governer of a certain garrison, being desirous to know how much money the passage of the garrison amounted to in a certain time, made choice of a loyal servant, giving him orders to receive of every coachman 4 cts., of every horseman 2 cts., and of every footman $\frac{1}{2}$ a cent. Now, at the end of the term, the servant gave the governor $227,50, and let him know that as often as 5 passed in coaches, 9 passed on horseback, and as often as 6 passed on horseback, 10 passed on foot.

Required, the number of passengers of each kind.

* First, if 6 : 10 : : 9 : 15 passed on foot as often as 9 passed on horseback, and 5 passed in coaches; hence their numbers were in proportion to each other as 5, 9, 15; and 5 coachmen paid 5 times 4 cts.=20 cts.; 9 horsemen paid 9 times 2 cts. =18 cts.; and 15 footmen paid 15 times $\frac{1}{2}$ a cent=$7\frac{1}{2}$ cts. Therefore, the sums which they paid, were in proportion to each other, as 20, 18, $7\frac{1}{2}$, the sum of which is $45\frac{1}{2}$; consequently, the coachmen paid $\frac{40}{91}$, the horsemen paid $\frac{36}{91}$, and the footmen $\frac{15}{91}$.

Then, if 91 : 227,50 : : 40 : $100, all the coachmen paid, which divided by 4 cts.=2500 coaches, Ans.

Also, 91 : 227,50 : : 36 : $90, all the horsemen paid, which divided by 2 cts.=4500 horsemen, Ans.

And, 91 : 227,50 : : 15 : $37,50, all the footmen paid, which divided by $\frac{1}{2}$ a cent,=7500 footmen, Ans.

35. On a certain day, 20 farmers, 30 merchants, 24 lawyers, and 24 tailors, spent at a dinner $64, which was divided among them in such a manner, that 4 farmers paid as much as 5 merchants, 10 merchants paid as much as 16 lawyers, and 8 lawyers as much as 12 tailors. How much money did each class pay?

* First, the number of each class which paid the same, was in proportion to each other as 4, 5, 8, 12; and as often as 4 farmers paid $1, 1 farmer paid $\frac{1}{4}$ of a dollar, and 20 farmers paid $\frac{20}{4}$=$5; and 4 farmers paid $1, and 5 merchants the same; therefore, 30 merchants paid $\frac{30}{5}$=$6.

And 5 merchants=8 lawyers, hence 24 lawyers=$\frac{24}{8}$=$3.
And 8 lawyers=12 tailors; therefore, 24 tailors=$\frac{24}{12}$=$2.

Hence the sums which they paid, were in proportion to each other as 5, 6, 3, 2, and the sum of these numbers is 16; therefore, the farmers paid $\frac{5}{16}$, the merchants $\frac{6}{16}$, the lawyers $\frac{3}{16}$, and the tailors $\frac{2}{16}$.

$$\text{Then, if}\left\{\begin{array}{llll} 16:64::5:\$20, & \text{what the} & \text{farmers} & \text{paid.} \\ \text{``}:\text{``}::6:\$24, & \text{``} & \text{merchants} & \text{``} \\ \text{``}:\text{``}::3:\$12, & \text{``} & \text{lawyers} & \text{``} \\ \text{``}:\text{``}::2:\$\ 8, & \text{``} & \text{tailors} & \text{``} \end{array}\right\}\text{Ans.}$$

36. If the earth turns upon its axis once in 23 hours, 56m., 4s., and if its circumference is 25000 miles, at what rate per hour are the inhabitants of the equator carried?

* Thus, 23h. 56m. 4s. : 25000 : : 1h. : $1044\frac{11196}{21541}$ miles, Ans.

37. If the sun move, every day, one degree, and the moon thirteen, and at a certain time the sun be at the beginning of Cancer, and in three days after, the moon at the beginning of Aries, the place of their next following conjunction is required.

* There are 30° in a sign, and the signs Taurus and Gemini occur between Aries and Cancer; and in 3 days after, the moon is at the beginning of Aries; therefore, the moon wanted 39° of Aries; hence, 129° between the two spheres' motions; and 13—1=12° per day, one sphere gains upon the other, and 129° degrees to be gained.

Hence, as 12 : 1 : : 129° : 10° 45′ of Cancer, Ans.

38. Suppose a meteor to move parallel to the earth's surface, and 50 miles from it, at the rate of 20 miles per second. In what time would it move round the earth?

* Suppose the earth's diameter 7964 miles.

Then, 7964,+50×2=8064, diameter of the circle described by the meteor. Then, 20 : 8064 : : 3,1416 : 1266,693120, which is 21′ 6″ 41‴ 35⁗ 13′′′′′ 55′′′′′′, Ans.

39. If the mean diameter of the earth's annual path round the sun is 191263000 miles, required its mean motion per minute?

*First, *the path* ×3,1416=600871840, 8m., orbit's circum.

Then, as 365¼d. : 600871840,8 : 1 : 1142,44 miles, ans.

40. A and B travel the same direction for the same place, and travel together at B's uniform speed for 9 days, when A turned and went back at B's rate of speed. Then turns again and pursues B at the rate of 18 miles per day, and overtakes him in 22½ days. What is B's rate of speed per day?

*Now since they are 9 days in company at their outset, and A travels back, and B keeps on, it shows that B travels 18 days the longest in a right line; and since A overtakes B in 22½ days, it shows that 18×22½=405 miles that B travels, and 22½+18=40½ days in travelling it.

Then, if 40½ ; 405 : : 1 : 10 miles, Ans.

41. A and B are on a straight road and on opposite sides of a gate; A is distant from it 308 yards, B 277 yards; travelling each towards the gate. A walks 2⅓ yds., and B 2 yards

per second. How long must they walk to make their distances equal from the gate?

*First, $2\frac{1}{3}$—$2=\frac{1}{3}$ of a yard gained by A per second,—and 308—277=31 yds to be gained.

Then, if $\frac{1}{3}$: 1s. : : 31 : 1m. 33s., Ans.

42. A man is to travel 335 miles; at the expiration of 7 days he found that the distance which he had to travel was equal to $\frac{25}{42}$ of the distance which he had already travelled. How many miles per day did he travel?

*Thus, 42 : 7 : : 25 : $4\frac{1}{6}$, then $4\frac{1}{6}+7$)335(30m. per day, Ans.

43. A pedestrian, for a wager of $1000 having engaged to travel 17 miles in 1 h. $34\frac{1}{2}$ m. finished 10 miles in 1 h. 0 m. 30 s. and performed the task in 1h. 31m. It is required to know whether he is before or after time when he had finished 10 miles, and how much before time when he finished.

*First, 1h. $34\frac{1}{2}$m. : 17 : : 1h. 30s. : 10 miles and 1555 yards, then, 10m. 1555yds.—10m.=1555 yds., after time at the end of 10 miles; and 1h. $34\frac{1}{2}$m.—1h. 31m.=$3\frac{1}{2}$m. before time when he finished.

Then, if 1h. $34\frac{1}{2}$m. : 17 : : $3\frac{1}{2}$m. : 1108yds. before time.

44. There is an Island 50 miles in circumference, and 3 men start together to travel the same way around it. A travels 7 miles per day, B 8 and C 9. In what time will they be together again; how far and how many times round will each travel, and in what time will C catch A the 3d time?

*Thus, 50×7,+50×8,+50×9=1200, the dividend.

And 7+8+9=24, the divisor. Then, 24)1200(50 days, Ans. 1.

Then, { 50×7=350 miles A travels,
50×8=400 " B "
50×9=450 " C "

And, { 50)350(7 times round A travels
50)400(8 " " B "
50)450(9 " " C "

} before all are together again.

Lastly, 9—7=2 miles per day that C gains upon A.

Then, if 2m. : 1d. : : 50 : : 25,+50=75 days, to catch A the third time.

45. Suppose two steamboats to start at the same time from places 300 miles apart on the same river, the one proceeding up stream is retarded by the current 2 miles per hour, the other moving down stream is accelerated the same; in still water their engines propel them 8 miles per hour. Now how far from each starting place will the boats meet?

*First, both boats in motion give 16 miles per hour.

Then, $16 \div 2 = 8$, and $8+2=10$ miles, the speed of one; then, $8-2=6$ miles, speed of the other, per hour.

$$\text{Then, } \left\{ \begin{array}{l} 16 : 300 :: 10 :: 112\tfrac{1}{2} \text{ miles, one proceeded,} \\ 16 : 300 :: \ 6 :: 187\tfrac{1}{2} \quad \text{" the other "} \end{array} \right\} \text{Ans.}$$

46. A ship of war sailed with 650 men, and provision for a cruise of 15 months. At the end of 3 months she captured an enemy's vessel, and put 75 men on board of her. Five months after, she captured and sunk another vessel, and took on board the crew, consisting of 350 men. How long did the provision last from the commencement of the cruise?

*First, $15-3=12$; $650-75=575$, and $575+350=925$.

Then, if $650 : 12 :: 575 : 13\frac{1}{2}\frac{3}{3}$ months, the provision would last 575, after 75 men were put on board of the first prize.

Lastly, $13\frac{1}{2}\frac{3}{3}-5=8\frac{1}{2}\frac{3}{3}$; then, as $575 : 8\frac{1}{2}\frac{3}{3} :: 925 : 5\frac{1}{3}\frac{2}{7}$m. then, $3+5+5\frac{1}{3}\frac{2}{7}=13\frac{1}{3}\frac{2}{7}$ months, Ans.

47. A ship's crew of 300 men were so supplied with provision for 12 months, that each man was allowed 30 ounces per day; but after sailing 6 months, they find that it will take 9 months more to finish their voyage, and 50 of the crew have been lost. Required the daily allowance of each man for the last 9 months.

*If $6 : 30 :: 9 : 20$oz.; then, $300 : 20 :: 250 : 24$oz., Ans.

48. A, B, C and D gained \$12; A was to have $\frac{1}{3}$ of it, B $\frac{1}{4}$, C $\frac{1}{6}$, and D $\frac{1}{8}$. How much has each?

*First, $\frac{1}{3}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}=\frac{21}{24}$, but separately, $\frac{8}{21}$ $\frac{6}{21}$ $\frac{4}{21}$ $\frac{3}{21}$, *because* $\frac{21}{24}$ *is not* $\frac{1}{1}$.

$$\text{Then, if} \left\{ \begin{array}{l} 21 : 12 :: 8 : \$4,\ 571\tfrac{3}{7} \text{ belong to A,} \\ \text{" : " :: } 6 : \$3,\ 428\tfrac{4}{7} \quad \text{" " B,} \\ \text{" : " :: } 4 : \$2,\ 285\tfrac{5}{7} \quad \text{" " C,} \\ \text{" : " :: } 3 : \$1,\ 714\tfrac{2}{7} \quad \text{" " D,} \end{array} \right\} \text{Ans.}$$

49. A, B, and C are to share £100 in the proportion of $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ respectively, but C dying, it is required to divide the whole sum properly between the other two.

$$\text{*Thus, as } \tfrac{1}{3}+\tfrac{1}{4}+\tfrac{1}{5} : £100 :: \left\{ \begin{array}{l} \tfrac{1}{3} : 42\tfrac{26}{47} \text{ A's} \\ \tfrac{1}{4} : 31\tfrac{43}{47} \text{ B's} \\ \tfrac{1}{5} : 25\tfrac{25}{47} \text{ C's} \end{array} \right\} \text{Answers if C had lived.}$$

Hence, $\frac{1}{3}+\frac{1}{4} : 25\frac{25}{47} :: \frac{1}{3} : 14\frac{5}{8}\frac{8}{4}\frac{7}{7}$£, A's part of C's.

And $42\frac{26}{47}+14\frac{5}{8}\frac{8}{4}\frac{7}{7}=$£57 2s. $10\frac{1}{4}$d. A's share of the £100; then, £100—£57 2s. $10\frac{1}{4}$d.=£42 17s. $1\frac{3}{4}$d. B's share.

50. A gentleman bought a chaise, horse, and harness for

£100 ; the horse cost $\frac{1}{4}$ more than the harness, and the chaise $\frac{1}{3}$ more than the horse ; what was the price of each ?

*First, let the price of the harness consist of 12 parts, then the price of the horse would consist of 15 parts, and the chaise of 20 parts ; then, $12+15+20=47$ parts in all.

$$\text{Then, if}\left\{\begin{array}{l} 47 : 100 : : 12 : £25\frac{25}{47}, \text{ price of the harness,} \\ \text{“} : \text{“} : : 15 : £31\frac{43}{47}, \quad \text{“ “ “ horse,} \\ \text{“} : \text{“} : : 20 : £42\frac{26}{47}, \quad \text{“ “ “ chaise,} \end{array}\right\} \text{Ans.}$$

51. A certain sum of money is to be divided between 4 persons in such a manner that the first shall have $\frac{1}{3}$ of it, the second $\frac{1}{4}$, the third $\frac{1}{6}$, and the fourth the remainder, which is $28. What is the sum ?

*First, $\frac{1}{3}+\frac{1}{4}+\frac{1}{6}=\frac{9}{12}$, hence $\$28=\frac{3}{12}$ of the required sum. Then, if $3:28::12:\$112$, Ans.

52. A received $\frac{1}{6}$ of a legacy, B $\frac{1}{10}$, and C the remainder ; and when they had counted what they received, A had $80 more than B. How much did each receive ?

*First, A's share is $\frac{5}{30}$, B's $\frac{3}{30}$, C's $\frac{22}{30}$; and $\frac{5}{30}-\frac{3}{30}=\frac{2}{30}$, $\frac{2}{30}=\$80$, and $\frac{30}{30}=\$1200$.

$$\text{Then, if}\left\{\begin{array}{l} 30 : 1200 : : 5 : \$200, \text{ A's share of the legacy,} \\ \text{“} : \text{“} : : 3 : \$120, \text{ B's “ “ “ “} \\ \text{“} : \text{“} : : 22 : \$880, \text{ C's “ “ “ “} \end{array}\right\} \text{Ans.}$$

53. A, B, C, D and E gained $4050, which was so shared that $\frac{1}{2}$ of A's part is equal to $\frac{1}{4}$ of B's, $\frac{1}{5}$ of C's, $\frac{1}{6}$ of D's and $\frac{1}{8}$ of E's. How much of the gain belonged to each ?

*First, consider B to have 4 shares, C 5, D 6, and E 8 ; then, A would have 8, and $8+4+5+6+8=31$ shares in all.

$$\text{Then, if}\left\{\begin{array}{l} 31 : 4050 : : 8 : \$1045\frac{5}{31}, \text{ belonged to A,} \\ \text{“} : \text{“} : : 4 : \$522\frac{18}{31}, \quad \text{“ “ B,} \\ \text{“} : \text{“} : : 5 : \$653\frac{7}{31}, \quad \text{“ “ C,} \\ \text{“} : \text{“} : : 6 : \$783\frac{27}{31}, \quad \text{“ “ D,} \\ \text{“} : \text{“} : : 8 : \$1045\frac{5}{31}, \quad \text{“ “ E,} \end{array}\right\} \text{Ans.}$$

54. A owns $\frac{7}{8}$ of an estate, and B the remainder ; A sells $\frac{3}{4}$ of his part to C, and C sells $\frac{3}{8}$ of his part to D, who also buys $\frac{1}{2}$ of A's remaining part. Required the share of each in the estate, and how much they would severally receive on selling it for $14720 ?

*First, B's share is $\frac{1}{8}$ of the estate ; A retains $\frac{11}{64}$ of his part of the estate, and C retains but $\frac{193}{256}$ of what he receives of A, and D receives $\frac{63}{256}$ of it from C, hence A's share is $\frac{44}{376}$, B's$=\frac{32}{376}$, C's$=\frac{193}{376}$, and D's$=\frac{107}{376}$.

Then, if $\left\{\begin{array}{l} 376 : 14720 :: \ \ 44 : \$1722\tfrac{208}{376} \text{ A receives,} \\ \text{“} \ : \ \text{“} \ \ :: \ \ 32 : \$1252\tfrac{288}{376} \text{ B} \quad \text{“} \\ \text{“} \ : \ \text{“} \ \ :: 193 : \$7555\tfrac{280}{376} \text{ C} \quad \text{“} \\ \text{“} \ : \ \text{“} \ \ :: 107 : \$4188\tfrac{352}{376} \text{ D} \quad \text{“} \end{array}\right\}$ Ans.

55. A younger brother received \$2200, which was just $\frac{5}{12}$ of his elder brother's fortune; and $3\frac{1}{8}$ times the elder's money was $\frac{1}{2}$ as much again as the father was worth. What was the father worth?

*First, if 5 : 2200 : : 12 : \$5280, which plus $3\frac{1}{8}$ of itself is equal to \$16500 = $1\frac{1}{2}$ the father's fortune. Ans., \$11000.

56. B delivers to C \$1200 to be invested in trade, on condition that if C added \$500 to it, and gave his time as manager he should have $\frac{2}{5}$ of the gain; what was C's time valued at?

*First, if C's time and \$500 was equal to $\frac{2}{5}$, \$1200 = $\frac{3}{5}$.
Hence, if 3 : 12 : : 2 : \$800, then 800 — 500 = \$300, Ans.

57. A certain gentleman at the time of marriage agreed to give his wife $\frac{2}{3}$ of his estate, if at the time of his decease he left only a daughter, and if he left only a son, she should have $\frac{1}{3}$ of his property; but, as it happened, he left a son and a daughter, by which the widow lost in equity \$2400 more than if there had been only a daughter. What would have been his wife's dowry if he had left only a son?

*As there is a son and a daughter, the son will have $\frac{4}{7}$ of the estate, the wife $\frac{2}{7}$, and the daughter $\frac{1}{7}$. If there had been only a daughter, her share would have been $\frac{2}{3}$; consequently she loses $\frac{2}{3} - \frac{2}{7} = \frac{8}{21}$.
Hence, if $\frac{8}{21} : \frac{7}{21}$: : \$2400 : \$2100, Ans.

58. A father devised $\frac{7}{18}$ of his estate to one of his sons, and $\frac{7}{18}$ of the remainder to the other, and the residue to his wife. The difference of the son's legacies was £257 3s. 4d. What money did he leave for his widow?

*If the first man's share be subtracted from the whole, there will remain $\frac{18}{18} - \frac{7}{18} = \frac{11}{18}$; and $\frac{7}{18}$ of $\frac{11}{18} = \frac{77}{324}$, the second son's share. And $\frac{7}{18} - \frac{77}{324} = \frac{49}{324}$, difference of their legacies; and $\frac{7}{18} = \frac{126}{324}$; then, $\frac{126}{324} + \frac{77}{324} = \frac{203}{324}$, legacy of both sons. Hence, $\frac{324}{324} - \frac{203}{324} = \frac{121}{324}$, wife's legacy.
Then, if $\frac{49}{324}$: £257 3s. 4d. : : $\frac{121}{324}$: £635 0s. $10\frac{30}{49}$d, Ans.

59. If A can do as much work in 3 days as B can do in $4\frac{1}{2}$ days, and B as much in 9 days as C in 12 days, and C as much in 10 days as D in 8 days, how many days work of D are equal to 5 days work of A?

I. This and the following question are *Conjoined Proportionals*, and the operations of such are performed by cancelling equal quantities on both sides, and abbreviating commensurables.

II. The first numbers in each part of the question are called *antecedents*, and the following *consequents*.

III. A little consideration will discover the difference in the *Operation* of each, and others of the same nature may be treated accordingly.

*First, $\left\{\begin{array}{l} 3 \times 9 \times 10 = 270, \\ 4\frac{1}{2} \times 12 \times 8 = 432, \end{array}\right.$ Then, if 270 : 432 : : 5 : 8 d., Ans.

60. If 25 pears can be bought for 10 lemons, and 28 lemons for 18 pomegranates, and 1 pomegranate for 48 almonds, and 50 almonds for 70 chestnuts, and 108 chestnuts, for $2\frac{1}{2}$ cents, how many pears can be bought for $1,35 cents?

*First, $\left\{\begin{array}{l} 25 \times 28 \times 1 \times 50 \times 108 = 3780000. \\ 10 \times 18 \times 48 \times 70 \times 2\frac{1}{2} = 1512000. \end{array}\right.$

Then, if 1512000 : 3780000 : : 1,35 : $337\frac{1}{2}$ pears, Ans.

61. A shepherd being asked how many sheep he had, said, 'if I had as many, half as many, and $\frac{1}{4}$ as many more, I should have 264.' How many had he?

*First, what he had was $\frac{4}{4}$; then, as many, $\frac{1}{2}$ as many, and $\frac{1}{4}$ as many, is $\frac{7}{4}$, and $\frac{4}{4}+\frac{7}{4}=\frac{11}{4}$, $=264$.

Then, if 11 : 264 : : 4 : 96 sheep, Ans.

62. A man that was feeding his geese, was accosted by another with "*Good morning*, with your hundred geese." He replied—I have not a hundred, but if I had half as many more as I now have, and two geese and a half, I should have a hundred. How many had he?

*First, $100-2\frac{1}{2}=97\frac{1}{2}$, and $97\frac{1}{2}$, $=3$ halves.

Hence, as 3 : $97\frac{1}{2}$: : 2 : 65 geese, Ans.

63. If 18 grains of silver will make a thimble, and 12dwt. a teaspoon, how many thimbles and teaspoons, of each an equal number, can be made from 15oz. 6dwt.?

*First, one thimble and one teaspoon$=$306 grains, and 15oz. 6dwt.$=$7344 grains; then, as 306 : 1 : : 7344 : 24, Ans.

64. Three men met at an Inn, two of them brought provision with them; but the third not having any, proposed to the other two that they should eat together, and he would pay them for his proportion. This being agreed to, A produced 5 loaves,

and B 3 loaves, which they eat together, and C paid 8 equal pieces of money as the value of his share; how much of the pay should A and B severally have?

*First, $3+5=8$ loaves; then, if $8:3$ men $::3$ loaves $:1\frac{1}{8}$, therefore, B produced $\frac{1}{8}$ more than he eat, and each eat $\frac{8}{8}$; hence, B should have $\frac{1}{8}$ of the money which is 1 piece, and A the remainder.

65. If 20 feet of iron railing weigh 1000lbs. when the bars are $1\frac{1}{4}$ inch square, what will 50 feet come to at $9\frac{3}{4}$ cts. per lb. when the bars are $\frac{7}{8}$ of an inch square?

*First, $50=20\times2\frac{1}{2}$; then, $1\frac{1}{4}\times8=10$, and $10\times10=\frac{100}{8}$, $\frac{7}{8}\times\frac{7}{8}=\frac{49}{8}$; then, if $100:1000:49::490$lbs.,$=20$ft. of the $\frac{7}{8}$ inch square; then, $490\times2\frac{1}{2}=1225$lbs.; then, 1225lbs.$\times9\frac{3}{4}$ cts.$=\$119,4375$, Ans.

66. A bought C's drove of swine, giving 18s. for hogs, 16s. for sows, and 2s. for pigs; there were as many hogs as sows, and for every sow there were 3 pigs, worth in all £50. How many was there of each?

*Thus, 18s.+16s.+6s.=40s., cost of 5 swine.

Then, if 40s. : 5 : £50 : 125 swine in all.

And if $5:125::3:75$ pigs, then $125-75=50$ hogs and sows, and $50\div2=25$ of either.

67. A with an intention to clear 30 guineas, (22s. 9d. each) on a bargain with B, rates hops at 16d. per lb. that cost him but 10d. per lb. B apprised of this, set down malt, that cost him 10s. 10d. per barrel, at an adequate price. How much malt did they contract for?

*First, if 10d. : 16 : 10s. 10d. : $17\frac{1}{3}$s., then $17\frac{1}{3}$s.—10s. 10d. $=6\frac{1}{2}$s. gain per barrel to be adequate to A's barter price, and as many barrels are contracted for as $6\frac{1}{2}$s. is contained times in the 30 guineas; hence 30 guineas$\div6\frac{1}{2}$s.$=105$ barrels, Ans.

68. A man lays out 30 cents for apples and pears, buying his apples at 4, and his pears at 5 for a cent, and afterwards sold $\frac{1}{2}$ of his apples, and $\frac{1}{3}$ of his pears for 13 cents, which was the prime cost. I demand the number he bought of each?

*If $\frac{1}{2}+\frac{1}{3}:13::\frac{1}{2}+\frac{1}{3}\times2:26$ cts., then $30-26=4$cts. that $\frac{1}{3}$ of the pears cost; then $\frac{3}{3}=12$cts., paid for pears; then $12\times5=60$ pears bought; then $30-12=18$, and 18cts.$\times4=72$ apples.

69. A person found two sums of money; $\frac{1}{4}$ of the first added to $\frac{1}{3}$ of the second was \$120, the two sums together were \$400. What was each sum?

*Thus, $\frac{1}{4}+\frac{1}{3}:120::\frac{3}{4}+\frac{2}{3}:\$360$, and $400-360=\$40=\frac{1}{4}$ of the first sum; then $40\times4=\$160$, the first sum.

Lastly, $400-160=\$240$, the second sum.

70. A gentleman had £7 17s. 6d. to pay among his laborers; to every boy he paid 6d., to every woman 8d., and to every man 16d.; there was 1 boy to 3 women, 1 woman to 2 men. What was the number of each?

$$\text{*First, } \frac{1 \text{ boy,}}{6\text{d.}} + \frac{3 \text{ women,}}{24\text{d.}} + \frac{6 \text{ men,}}{96\text{d.}} = \frac{10 \text{ persons.}}{126\text{d.}}$$

Then, if 126d. : 10 persons, : : £7 17s. 6. : 150 persons in all; and, if 10 : 150 : : 1 : 15 boys, then $15\times3=45$ women, then $45\times2=90$ men.

71. Sold a piece of cloth for 850 guineas, and gained upon every yard $\frac{1}{8}$ the prime cost of an ell English. Required the cost of the piece.

$$\text{*}\frac{\text{1 ell Eng.}=5\text{qr.}}{\text{1 yard}\ =4\text{qr.}}\times\frac{1}{8}=\frac{5,}{32}+32=\frac{37,}{32,}\ \begin{matrix}\text{cost and gain.}\\ \text{cost.}\end{matrix}$$

That is, 850 guineas=£$892\frac{1}{2}=\frac{37}{32}$.

Then, if 37 : $892\frac{1}{2}$: : 32 : £771 17s. $10\frac{2}{37}$d. cost, Ans.

72. A paid \$50 for linen and cotton cloth, paying a dollar for 3 yards of linen, the same for 5 yards of cotton; he afterwards sold $\frac{1}{4}$ of his linen and $\frac{1}{5}$ of his cotton for \$12, which was 60 cents more than it cost; how much of each did he purchase?

*First, $\$12-60\text{cts.}=\$11,40$, cost of $\frac{1}{4}$, and $\frac{1}{5}$.

Then, if $\frac{1}{4}+\frac{1}{5}:11,4::\frac{4}{4}+\frac{4}{5}:\$45,60$, cost of the linen and $\frac{4}{5}$ of the cotton; then, $\$50-\$45,60=\$4,40$, price of the cotton's $\frac{1}{5}$ part; hence $\$4,40\times5=\$22$, paid for cotton, and \$50 less $\$22\neq\$28$, paid for linen; then, $\$22\times5=110$ yds. of cotton; and $\$28\times3=84$yds. of linen.

73. Bought a quantity of linen, at 12s. 8d, for 3 yds., and by selling it at 32s. 8d. for 7 yds., I gained as much as 24 yds. cost. Required, the number of yards bought.

* First, if 3 : 12s. 8d. : : 7 : £1 9s. $6\frac{2}{3}$d.; then $32\frac{2}{3}$s.—1£ 9s. $6\frac{2}{3}$d.=3s. $2\frac{1}{3}$d. If 3s. $2\frac{1}{3}$d. : 7 : : 4s. $2\frac{2}{3}$d $\times$ 24 : 224 yds., Ans.

74. Laid out in a lot of muslin, £500, upon examination of which, 3 parts in 9 proved damaged, so that I could make but 5s. a yard of the same; and by so doing, find I lost £50 by it. At what rate per ell Eng. am I to part with the undamaged muslin, in order to gain £50 upon the whole?

* First, if 3 : £50 : : 9 : £150, loss on £500, at £50 per 3 parts, then 500—150=£350 that 9 parts cost, at 5s. a yard.

Then, £350×20s.÷5s.=1400 yards in all.

And, if 9 : 1400 : : 3 : 466$\frac{2}{3}$ yards of damaged muslin.

Then, 1400—466$\frac{2}{3}$=933$\frac{1}{3}$ yards undamaged.

Also, 933$\frac{1}{3}$×4,÷5=746$\frac{2}{3}$ ells Eng.

Then, 466$\frac{2}{3}$×5s.=£116 13$\frac{1}{3}$s., cost of 3 parts at 5s. a yard; then, £500+£50,—£116 13$\frac{1}{3}$s.=£433 6$\frac{2}{3}$s., cost of the 6 undamaged parts.

And £433 6$\frac{2}{3}$s.÷746$\frac{2}{3}$=11s. 7$\frac{2}{7}$d. per ell Eng., Ans.

75. A gay young fellow soon got the better of $\frac{2}{7}$ of his fortune; he then gave $1500 for a commission, and his profusion continued till he had but $450 left, which he found to be just $\frac{3}{8}$ of his money, after the purchase of his commission. What was his fortune at first?

* Thus, as 3 : 450 : : 8 : 1200, and 1200+1500=$2700, equal $\frac{5}{7}$ of his fortune; then, as 5 : 2700 : : 7 : $3780, Ans.

76. A, B, C, and D spend 35s. at a reckoning, and being a little dipped, agreed that A should pay $\frac{2}{3}$, B $\frac{1}{2}$, C $\frac{1}{3}$, and D $\frac{1}{4}$. Now, what should each pay?

* Thus, $\frac{2}{3}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}=\frac{21}{12}$, and severally, $\frac{8}{21}$ $\frac{6}{21}$ $\frac{4}{21}$ $\frac{3}{21}$, for $\frac{21}{12}=1\frac{1}{12}=\frac{1}{1}$ plus $\frac{1}{12}$.

$$\text{Then, as}\left\{\begin{array}{l} 21 : 35\text{s.} : : 8 : 13\text{s. 4d. A should pay.} \\ \text{“} : \text{“} : : 6 : 10\text{s. 0d. B “} \\ \text{“} : \text{“} : : 4 : \ \ 6\text{s. 8d. C “} \\ \text{“} : \text{“} : : 3 : \ \ 5\text{s. 0d. D “} \end{array}\right\}\text{Ans.}$$

77. A person having spent in one year all his income, and $\frac{1}{4}$ as much more, found that by saving $\frac{1}{15}$ of his income afterward, he could make good the deficiency in 4 yrs., and have $20 left. Required, his income.

* Now, $\frac{1}{15}$ of his annual income, for 4 years, is $\frac{4}{15}$ of it for 1 year; and $\frac{4}{15}$ of 1 year's income, is $20 more than $\frac{1}{4}$ of it; therefore, $20 is $\frac{1}{60}$ of his income.

Hence, if 60 : 20 : : 60×60 : $1200, Ans.

78. A merchant begins the world with $1500, and finds that by his distillery, he clears $1500 in 7 years; by his navigation, $1500 in 9 years; and that he spent in gaming, $1500 in 3$\frac{1}{2}$ years. How long will his estate last?

$$*\ \text{If}\left\{\begin{array}{l} 7 : 1500 : : 1 : \$214\tfrac{2}{7} \\ 9 : \text{“} : : 1 : \$166\tfrac{2}{3} \\ \hline 3\tfrac{1}{2} : \text{“} : : 1 : \$428\tfrac{4}{7} \end{array}\right\}$$

And 214$\frac{2}{7}$ + 166$\frac{2}{3}$ = 380\frac{20}{21}$, gain, and 428$\frac{4}{7}$ — 380$\frac{20}{21}$ = 47\frac{91}{147}$, spent more than his income.

Then, if 47$\frac{91}{147}$: 1yr. : : 1500 : 31$\frac{1}{2}$ years, Ans.

79. A father dying, left his son a fortune, $\frac{1}{4}$ of which he spent in 8 months; $\frac{3}{7}$ of the remainder lasted him 12 months longer, after which he had only $1200 left. How much did his father leave to him?

* After spending $\frac{1}{4}$, the remainder is $\frac{3}{4}$.

Hence, if 4 : 1200 : : 7 : $2100, left after spending $\frac{1}{4}$ of the fortune; then, if 3 : 2100 : : 4 : 2800, Ans.

80. The sides of two square pieces of ground are as 3 to 5, and the sum of their superficial contents is 30600 square feet. What is the length of a side of each piece?

* Thus, $3\times3=9$; then $5\times5=25$; and $25+9=34$; and as 34 : 30600 : : 9 : 8100, and $\sqrt{8100}=90$ feet, side of the smaller piece; then, if 34 : 30600 : : 25 : 2250, and $\sqrt{2250}=$ 150 feet, side of the greater.

81. G and H buy 48 acres of land, at $12 per acre, of which H is to have a piece containing 12 acres, which G and H think to be $\frac{1}{3}$ better than 12 of the 36 acres that G is to have. How much should each pay?

* Since G's land is $\frac{3}{4}$ of the whole, he receives $\frac{3}{4}$ of the $\frac{1}{3}$ better; and $48\times12=\$576$, the land cost; and $\frac{1}{3}$ of the cost of 1 acre, is $4, and $12\times4=\$48$, $\frac{3}{4}$ of which is paid by H; also, $12\times12=\$144,+36=\$180$, paid by H; then, if 48 : 576 : : 36 : 432, then $432-36=\$396$, paid by G.

82. A and B purchased 20 acres and 72 rods of land, at $25, 57 cts. per acre. By agreement, A was to have 6 acres and 12 rods of the land, and B the remainder, and pay $4 per acre more than A. How much did each man's land cost him per acre?

* $25, 57 cts.$\times$20,45=$522,906, cost of the land; and acres 20,45—6,075=14, 375 acres, belong to B; and 14,375 $\times$$4=$57,50 cts., extra price of B's land; then, 522,906—57,5=$465,406+, and 465,406$\div$20,45=$22,75 cts., 8 m., A's land cost him per acre. 22,758+4=$26,75 cts., 8 m., B's land cost him per acre.

83. B and C bought 1200 acres of land, at $1 per acre, each paying $600. Sometime after, C on viewing it, offers to take a certain square piece at $1,75 cts. per acre, to the amount of his advance, to which B consents. How many acres will each have? What is the length of each side of C's lot, and what does B's land cost him per acre?

* First, if $1,75 : 1 : : 600 : 342 acres, 3 roods, 17$\frac{1}{7}$ rods, C

has, and 1200 acres less C's part$=$857 acres, $22\frac{6}{7}$ rods B has; and 857 acres $22\frac{6}{7}$ rods : \$600 : : 1 : 70 cts. per acre, B's; $\sqrt{342}$ acres 3 roods, $17\frac{1}{7}$ rods,$=$234 rods 3 ft. $6\frac{3}{4}$ inches, length of either side of C's lot.

REMARKS.—Most of the preceding and following questions are of different natures, and the solution of each may be used as a model for solving all others of the same nature.

Teachers having pupils to instruct in Arithmetic, would do well to class them, as in reading; and after placing the solution of these questions upon the Black-board, they should be required to ask each other the reason of each step in the operation. This will enable teachers to be more thorough in giving their instructions, and will incite pupils to greater diligence.

PERCENTAGE PROPORTIONALS,

AND SUCH INTERESTING AND INTRICATE QUESTIONS AS OFTEN OCCUR IN SCHOOLS, RANKED IN CLASSES.

REMARK.—Teachers of common schools, *en masse*, readily agree, that matter of an intricate nature, in the various works on Arithmetic, has ever been a greater check to the progress of pupils in this science, than parents and guardians generally are aware. And since the preceding and following questions are placed within the comprehension of the common scholar, it is presumed they must be interested with, and immediately engage in the study of what were once objects of dread, but now rendered subjects of pleasure and profit, by which principles are illustrated; and hence, the best source for obtaining a practical knowledge of figures; for, if pupils can thoroughly analyze intricate questions, in all their variety, it is presumed they will readily perform all practical questions.

I. LOSS AND GAIN.

CLASS I.

Required to find the gain or loss per cent.

EXAMPLES.

1. A stationer sold quills at \$1,83$\frac{1}{3}$ cts. per thousand, by

which he cleared $\frac{3}{8}$ of the money; but, as they grew scarcer, he raised them to \$2,25 cts. per thousand. What did he clear per cent. by the latter price?

* First, $183\frac{1}{3}\times3\div3=68\frac{3}{4}$ cts., first gain; hence, \$1,14$\frac{7}{12}$, first cost; then, $225-114\frac{7}{12}=$ \$1,10$\frac{5}{12}$ cts., last gain per thousand.

Then, if \$1,14$\frac{7}{12}$: \$1,10$\frac{5}{12}$: : \$100 : \$96,36$\frac{4}{11}$ cts., Ans.

2. A bought of B a lot of lamb's wool, washed, weighing 2720 lbs., at 46$\frac{1}{2}$ cts per lb.; but thinking it was not sufficiently dry, he takes it only on condition that a parcel be dried, and by that the state of the whole be determined. On trial, the loss was ascertained, and deduction made as stipulated, when A paid B \$1169,94 cts. in full. Required, the percentage lost by drying.

* Thus, $2720 \times 46\frac{1}{2} =$ \$1264,80 cts., its cost when not dried; then, 1264,80—1169,94=\$94,86 cts., its loss by being dried.

Then, if 1264,80 : 94,86 : : 100 : 7$\frac{1}{2}$ per cent, Ans.

3. What would be the duty on a piece of flannel 30 yards long and 42 inches wide, if it was estimated at 40 cents per square yard, and 30 per cent. *ad valorem*,* and what percentage would it pay on the original cost, if charged at 15d. sterling per yard?

* First, 15d. sterling=27 cts. 8m., and 30 yards long, and 42 inches wide=35 square yards of flannel; then, \$0,278×35 =\$9,73 cts., its extra cost, and cts. 40×35 yds.=\$14, its cost, and 14+9,73=\$23,73 cts., and 23,73÷35=\$0,6762.

Then, if $\begin{cases} \text{40 cts. : 30 per cent. : : \$0,6762 : 50 per cent., Ans.} \\ \text{\$100 : 30 : : \$14 : \$4,20 cts., duty, Ans.} \end{cases}$

4. Bought 30 hhds. of molasses, for \$600; paid in duties, \$20,66 cts.; Freight, \$40,78 cts.; storage, \$6,5 cts., and insurance, \$30,84 cts. If I sell it at \$26 per hhd., what will be gained per cent?

* Its cost and expense, \$698,33 cts.; it sold for \$780, and 698,33—780=\$81,67 cts., gained.

Then, as \$698,33 cts. : \$81,67 cts. : : \$100 : \$11,69+, Ans.

5. A house completely finished, cost the owner \$12894; it is 4 stories high, and the ground floor is divided into two shops, one of which is let at \$225, the other at \$200 a year; the 3

* *Ad valorem* is a Latin word, which signifies *according to value.*

upper stories are let for $450 a year, the annual expense for repairs, is $36,89 cts. What per cent. does the house pay?

* The rent is $875, and 875—36,89=$838,11 cts.

Then, if $12894 : $838,11 : : $100 : $6,50 per cent., Ans.

6. A bought flour for cash, and sold it to B at an advance. B sold it to C at 10 per cent., advance, and C sold it to D at $71,28 cts. advance, equal to 12 per cent. profit, which was 4 per cent. more than A made, though he bought it at $5 per barrel. Required, B's gain, how much C received, and the number of barrels in the lot?

* Since C gained 12 per cent., which was 4 per cent. more than A made, it shows that A gained 8 per cent., and sold it to B at $5,40 cts. per barrel, and B sold it to C at 10 per cent. profit; hence C paid $5,94 cts. per barrel. Then, if $1, : 12 cts. : : $5,94 cts. : $0,7128, C's gain per barrel, on selling it to D. And, if $0,7128 : 1 bbl. : : $71,28 cts. : 100 bbls., Ans. 3. And since it cost C $5,94 cts., and he gained $0,7128, it stood D $6,6528 per barrel; and $665,28 cts. per 100 barrels, Ans. 2. And since it stood B $5,40 cts., and C $5,94 cts. per barrel, say 5,94—5,40=54 cts., B gained per barrel; hence, $54 per hund. bbls., Ans. 1.

CLASS II.

Required to know how an article must be sold to gain or lose so much per cent.

EXAMPLES.

1. Bought a pipe of wine, containing 126 gallons, at 10s. per gallon, but, by accident, 16 gallons leaked out. At what rate must the remainder be sold per gallon, to gain on the whole prime cost, at the rate of 12½ per cent?

* First, 126 gallons, worth $210, and 126—16=110 gallons not leaked out; then, if 100 : 112,5 : : 210 : $236,25 cts., what it must sell for, and 236,25÷110=$2,14 cts. 7m. per gallon, Ans.

2. B imported 10 tons of iron, at $95 per ton, sundry expenses were $170. At what rate per lb. must he sell it to gain 20 per cent.?

* Cost and expense, $1120, and 10 T.=22400 lbs.

Then, if 100 : 120 : : 1120 : $1344, and $1344÷22400=6 cents per pound, Ans.

3. How should fish be rated, which is now worth in Boston 15s. per quintal, so that by selling it on 9 months credit, there may be 15 per cent. gain, after allowing 5 per cent. discount for present payment?

* As 100 : 100+15+6 : : 15 : 18.15s.

Interest of 100 at 5 per cent. for 9 months, is 3.75.

Hence, as 100 : 3,75 : : 18.15 : ,68025s. discount.

Then, 18.15+,68025=18.830625s., Ans.

CLASS III.

Required the cost of an article, when there is gained or lost so much per cent.

EXAMPLES.

1. A merchant receives three kinds of flour; from A he receives 20 barrels, from B 25 barrels, and from C 40 barrels. He finds that A's flour is 10 per cent. better than B's, and that B's is 20 per cent. better than C's. He sells the whole at $6 per barrel. What in justice should each man receive?

* A 20×$132=$2640	10 per cent. on $120=$132, 85 bbls.×$6=$510, it sold for?
B 25×$120=$3000	
C 40×$100=$4000	
85 bbls. $9640	

Then, as
9640 : $510 : : 2640 : 139\frac{161}{241}$, A receives.
" : " : : 3000 : 158\frac{172}{241}$, B " Ans.
" : " : : 4000 : 211\frac{149}{241}$, C "

2. A bought a house for $1575, and repaired it for D, who agreed to pay him a rent of $220 per annum, which was 12 per cent. for his money that he paid for the house and its repair. What was the cost of repairing it?

* If 12 : 100 : : 220 : 1833\frac{1}{3}$, which less 1575=$258$\frac{1}{3}$, Ans.

3. A father left two sons, the one 11 and the other 16 years of age, $10000, to be divided so that each share, being put to interest at 5 per cent., might amount to equal sums when they would be respectively 21 years of age. Required, the share of each?

* Now it is evident that the youngest son must have less than half the $10000, because his share will be at interest 10 years, and the share of the oldest only 5 years. And the amount of $1 for 10 years, at 5 per cent., is $1,50; and the

amount of $1 for 5 years, at 5 per cent., is $1,25. 1,50+1,25 is 2,75.

Then, $\left\{\begin{array}{l} 2,75 : 10000 : : 1,50 : \$5454\frac{6}{11}, \text{the older's share.} \\ \text{“} : \text{“} : : 1,25 : \$4545\frac{5}{11}, \text{the younger's.} \end{array}\right\}$ Ans.

4. A and B cleared by an adventure at sea, 45 guineas, which was 35 per cent. upon the money advanced, and with which they agreed to purchase a genteel horse and carriage, whereof they were to have the use in proportion to the sums adventured, which was found to be 11 to A, as often as 8 to B. How much did each adventure?

* Each guinea=£1$\frac{2}{5}$, and 45×1$\frac{2}{5}$=£63. 11+8=19.

And, if 35 ; 63 : : 100 : £180 adventured.

Then, if $\left\{\begin{array}{l} 19 : 180 : : 8 : £75\ 15s.\ 9\frac{9}{19}d., \text{B adventured.} \\ \text{“} : \text{“} : : 11 : £104\ 4s.\ 2\frac{10}{19}d., \text{A} \quad \text{“} \end{array}\right\}$ Ans.

5. A had 12 pipes of wine, which he parted with to B at 4½ per cent. profit, who sold them to C for $40,60 advantage; C made them over to D for $605,50, and cleared thereby 6 per cent. How much per gallon did this wine cost?

* 12 pipes=1512 gallons. If 106 : 100 : : 605,50 : $571,2264, which less $40,60=$530,6264, it cost B.

Then, if 104,5 : 100 : : 530,6264 : $510,1215.

And 510,1215÷1512=33$\frac{1116}{1512}$ cts., Ans.

6. G owns $\frac{2}{7}$ of $\frac{1}{2}$, and $\frac{3}{7}$ of $\frac{4}{11}$ of the other $\frac{1}{2}$ of an individual estate. Suppose this estate rents for $1556,94, being equal to 6 per cent. per annum on its value, and that he sells $\frac{1}{2}$ of his part on such terms as to yield the purchaser 8 per cent. per annum on his payment. How much does G receive, and what share has he now in the estate?

* First, $\frac{2}{7}$ of $\frac{1}{2}=\frac{1}{7}$, and $\frac{3}{7}$ of $\frac{4}{11}$ of $\frac{1}{2}=\frac{12}{154}$.

Then, $\frac{12}{154}+\frac{1}{7}=\frac{34}{154}$, and $\frac{34}{154}\div 2=\frac{17}{154}$, G owns of the estate.

And, if 154 : 1556,94 : : 17 : $171,87.

Then, if 8 : 100 : : 171,87 : $2148,37½, G receives, Ans. 1.

7. A, on preparing for a voyage to Calcutta, purchased of G, specie dollars, to be paid in 18 months, with interest. Supposing the premium on the dollars to be 3 per cent., and that G would have 5 per cent. per annum for the use of his money to be inserted in the note, which was given for $22145. I would know the sum purchased.

* The interest of 3 per cent., at 5 per cent., for 18 months, is ,225, and the interest of $100, for 18 months, at 5 per cent., is $7,50.

Then, if 100+7,5+3+,225 : 100 : : 22145 : \$20000, Ans.

8. Sold a horse for \$60, and by so doing lost 20 per cent., whereas, I ought to have gained 30 per cent. What was the real value of the horse?

* If 100—20 : 60 : : 100 : \$75.

And, if 100 : 130 : : 75 : \$97,50 cts., Ans.

9. B owns $\frac{3}{7}$ of a ship, which cost him \$3691, and sells $\frac{3}{8}$ of his part to C, at 12 per cent. advance. Required, the share of each in the ship, the cost of C's part, and how much B's part stands him.

* If 8 : 3691 : : 3 : \$1384,125=$\frac{3}{8}$ of \$3691.

And, if 100 : 112 : : 1384,125 : \$1550,22, C's part cost him. 1550,22—1384,125=166,095, and 3691—166,095=\$3524,-905, B's part stands him.

And C's part is $\frac{3}{8}$ of $\frac{3}{7}=\frac{9}{56}$. And B's part is $\frac{5}{8}$ of $\frac{3}{7}=\frac{15}{56}$.

10. A bought $\frac{25}{37}$ of a store, and soon after he paid \$1230,075, for $\frac{7}{37}$ of the same, which was at 10 per cent. advance on the rate of the first purchase; and to own the whole, he bought the remainder at 25 per cent. advance on it. What did the store cost him?

* If 110 : 100 : : 1230,075 : \$1118,25, first purchase cost of 7 shares.

And, if 7 : 1118,25 : : 25 : \$3993,75, cost of 25 shares.

Then, if 7 : 1118,25 : : 5 : \$798,75, first purchase cost of 5 shares.

And, if 100 : 125 : : 798,75 : \$998,4375, cost of 5 shares.

Then, 3993,75+1230,075+998,4375=\$6222,2625, Ans.

CLASS IV.

Required to know what would be gained or lost per cent. by goods sold at another rate, when there is gained or lost so much per cent. by being sold at a certain rate.

EXAMPLES.

1. If by selling goods at 2s. 3d. per lb., I clear cent per cent., what do I clear per cent. by selling them at 9 guineas per cwt.?

* 2s. 3d.=27d., 9 guineas is 2268d.; 2268÷112=20$\frac{1}{4}$d. per lb., and cent per cent. is 2 cents received for every cent laid out; then, if 27 : 200, cent per cent. : : 20$\frac{1}{4}$: 150, and 150—100=50 per cent., Ans.

2. If by selling goods at 2\frac{1}{2}$ per cwt. I gain 20 per cent., what do I gain or lose per cent., by selling them at 2\frac{1}{4}$ per cwt.?

* If 2,5 : 120 : : 2,25 : 108, and 108—100=8 per cent. gain, Ans.

3. If wine, sold at $1,50 per gallon, be 15 per cent. profit, what would be the gain or loss per cent. if sold at $1,25 per gallon?

* If 115 : 100 : : 150 : $1,30$\frac{10}{23}$, first cost per gallon.
And 130$\frac{10}{23}$—125=5$\frac{10}{23}$ cts., loss on its cost at the other rate.
Then, if 130$\frac{10}{23}$: 5$\frac{10}{23}$: : 100 : $4,16$\frac{2}{3}$, loss per cent., Ans.

4. Bought 126 gallons of rum, for $110. How much water must be added to reduce the first cost to 75 cts. per gallon?

* If 75 cts. buy 1 gallon, how many gallons will $110 buy? From this result, subtract the 126 gallons, and you have 20$\frac{2}{3}$ gallons, *the answer*.

5. Bought 115 gallons of wine, at $1,10 per gallon. How many gallons of water must be added, so as to gain $5 by selling it at $1 per gallon?

* 115×110, its cost, which plus $5=$131$\frac{1}{2}$.
Then, as $1 : 1 gal. : : 131\frac{1}{2}$: 131$\frac{1}{2}$ gallons of the mixture; and 131$\frac{1}{2}$—115=16$\frac{1}{2}$ gallons of water, Ans.

6. A merchant bought goods to the amount of $3472, which he sold at a loss of 12$\frac{1}{2}$ per cent., and invested the proceeds of the sale in other goods, which he sold at 13 per cent. profit. Did he gain or lose by these transactions, and how much per cent.?

* 100 : 100—12,5 : : 3472 : $3038 left.
And, if 100 : 113 : : 3038 : $3432,94, and 3472—3432,94 =$39,06, loss, and consequently, 1$\frac{31}{248}$ per cent., Ans.

II. PARTNERSHIP.

EXAMPLES.

1. A clears $13 in 6 months, B $18 in 5 m., and C $23 in 9 m., his stock being 72\frac{1}{2}$. What then is the general stock?

$$\begin{array}{l} \text{* First, as } 72,5 : \left.\begin{array}{r} 9\text{m.} : : 23 \\ 5\text{m.} : : 18 \end{array}\right\} : \$102,13\tfrac{1}{23}, \text{ B's stock.} \\ \hphantom{\text{* First, as } 72,5 : 9\text{m.} : : 23 \}} \ \$72,50, \quad \text{C's} \quad \text{"} \\ \text{Then, as } 102,13\tfrac{1}{23} : \left.\begin{array}{r} 5\text{m.} : : 18 \\ 6\text{m.} : : 13 \end{array}\right\} : \$61,46\tfrac{7982}{10800}, \text{ A's "} \\ \hline \hphantom{\text{Then, as } 102,13 : 5\text{m.} : : 18 \}} \ \$23609\tfrac{97193}{124200}, \text{ Ans.} \end{array}$$

2. A and B commence trade in company; A advanced at first $500, and B $750; at the end of 4 months, A adds $600 to his stock. What sum must B advance at the same time, to entitle him at the end of the year to equal profits with A?

$$\text{* First, } \left\{\begin{array}{l} 600\times 8=4800 \\ 500\times 12=6000 \\ 750\times 12=9000 \end{array}\right\} \text{ And } 6000+4800,-9000=1800.$$

$$\text{Then, if } \left\{\begin{array}{c} 10800 \\ 8 \end{array}\right\} : 1800 : : \left\{\begin{array}{c} 9000 \\ 12 \end{array}\right\} : \$225, \text{ Ans.}$$

PRINCIPLE.—*Hence it is evident, that when the times are equal, the shares of the gain or loss are as the stocks; and when the stocks are equal, the shares are as the times; but when neither are equal, the shares must be as their products.*

3. A, B, C, and D companied, and gained a sum of money, of which A, B, and C took $300; B, C, and D $400; C, D, and A, $380; and D, A, and B, $360. What was the gain of each?

*300+400+380+360,÷3=$480, the sum gained.

And 480—400=$80, A's gain; 480—380=$100, B's gain.

Then, 480—360=$120, C's gain; 480—300=$180, D's gain.

4. A and B commenced business with equal sums of money. A gained a sum equal to $\frac{1}{5}$ of his stock, but B lost $200, and then had only half as much as A. What was the original stock of each?

* The stock of each=$\frac{5}{5}$; $\frac{1}{5}$ of either's stock, and $200=$\frac{1}{2}$ as much as A's stock and gain, which is $\frac{6}{5}$; and B's stock less $200=$\frac{3}{5}$, hence $200=$\frac{2}{5}$ of either's stock, and consequently, $500 either's original stock.

5. A, B, and C put in $720, and gained $540, of which so often as A took $3, B took $5, and C $7. What did each put in and gain?

* Find a multiplicand to be multiplied by the given proportions, thus, 3+5+7)720(48, multiplicand to find the stocks.

And 3+5+7)540(36, multiplicand to find the gains.

$$\text{Then, }\left\{\begin{array}{l}48\times3=\$144,\text{ A's stock.}\\ \text{“}\times5=\$240,\text{ B's “}\\ \text{“}\times7=\$336,\text{ C's “}\end{array}\right.\text{ and }\left\{\begin{array}{l}36\times3=\$108,\text{A's gain}\\ \text{“}\times5=\$180,\text{B's “}\\ \text{“}\times7=\$252,\text{C's “}\end{array}\right.$$

6. A, B, and C companied, and put in together $1911. A's money was in 3 months, B's 5 m., and C's 7 m.; they gained $117, which was so divided as that $\frac{1}{2}$ of A's gain was equal to $\frac{1}{3}$ of B's, and $\frac{1}{4}$ of C's gain. What did each gain and put in?

* Suppose A's gain was $2, then B's must have been $3, and C's $4, by the question.

$$\text{Then, as }2+3+4:\$117::\left\{\begin{array}{l}2:\$26,\text{ A's gain.}\\ 3:\$39,\text{ B's “}\\ 4:\$52,\text{ C's “}\end{array}\right.$$

Then divide each man's gain by his time; and as the sum of these quotients is to each particular quotient, so is the whole stock to each man's particular stock.

7. C, D, and E company, and put in together $3860; C's money was in 3 months, D's was in 5 months, and E's was in 7 months; they gained $234, which was so divided, that $\frac{1}{2}$ of C's gain was equal to $\frac{1}{3}$ of D's, and $\frac{1}{3}$ of D's was equal to $\frac{1}{4}$ of E's. What did each put in and gain?

* First, allow C 2 shares, D 3 shares, and E 4 shares, the sum of which would be 9 shares in all.

$$\text{Then, as }\left\{\begin{array}{l}9:234::2:\$\ \ 52,\text{ C's gain.}\\ \text{“ : ” }::3:\$\ \ 78,\text{ D's “}\\ \text{“ : ” }::4:\$104,\text{ E's “}\end{array}\right\}\text{Ans.}$$

Then, $\$52\div3$ months$=17\frac{1}{3}$, and $\$78\div5$ months$=15\frac{3}{5}$, and $\$104\div7$ months$=14\frac{6}{7}$; then $17\frac{1}{3}+15\frac{3}{5}+14\frac{6}{7}=47\frac{83}{105}$.

$$\text{And, if}\left\{\begin{array}{l}47\frac{83}{105}:17\frac{1}{3}::3860:\$1400,\text{ C put in.}\\ \text{“ }:15\frac{3}{5}::\text{ “ }:\$1260,\text{ D “}\\ \text{“ }:14\frac{6}{7}::\text{ “ }:\$1200,\text{ E “}\end{array}\right\}\text{Ans.}$$

8. Three men engage in partnership for 20 months. A at first put into the firm $4000, and at the end of 4 months he put in $500 more; but at the end of 16 months, he took out $1000. B at first put in $3000, but at the end of 10 months, he took out $1500, and at the end of 14 months, he put in $3000. C at first put in 2000, and at the end of 6 months, he put in $2000 more, and at the end of 14 months, he put in $2000 more; but at the end of 16 months, he took out $1500. They gained by trade $4420. What is each man's share of the gain?

* $4000 for 4 months, is $100 for 160 months. $4000 + $500 = $4500 for 12 months, is $100 for 540 months; and $4500 — $1000 = $3500 for 4 months, is $100 for 140 months. A may therefore be considered as having $100 in trade, 160 + 540 + 140 = 840 months. Again, $3000 for 10 months, is $100 for 300 months. $3000 — $1500 = $1500 for 4 months, is $100 for 60 months. $1500 + $3000 = $4500 for 6 months, is $100 for 270 months. B may therefore be considered as having $100 in trade for 300 + 60 + 270 = 630 months. Again, $2000 for 6 months, is $100 for 120 months. $2000 + $2000 = $4000 for 8 months, is $100 for 320 months. $4000 + $2000 = $6000 for 2 months, is $100 for 120 months. $6000 — $1500 = $4500 for 4 months, is $100 for 180 months. C may therefore be considered as having $100 in trade for 120 + 320 + 120 + 180 = 740 months. The sum of A, B, and C's time is 840 + 630 + 740 = 2210 months. A's share of the gain will therefore be $\frac{840}{2210}$ of $4420; B's share, $\frac{630}{2210}$ of $4420; and C's share, $\frac{740}{2210}$ of $4420.

$$\text{Then, as}\left\{\begin{array}{l}2210 : \$4420 :: 840 : \$1680, \text{ A's gain.}\\ \text{“} \quad : \quad \text{“} \quad :: 630 : \$1260, \text{ B's “}\\ \text{“} \quad : \quad \text{“} \quad :: 740 : \$1480, \text{ C's “}\end{array}\right\}\text{Ans.}$$

9. A and B hire a coach in Portland, to go 72 miles, for $25, with the privilege of taking in three more persons. Having gone 20 miles, they take in C; on their return, they admit D; and within 30 miles of the city, they permit E to join them. How much shall each man pay?

* A and B had the use of the coach for the first 20 miles, for which they pay $\frac{20}{144}$ of $\$25 = \$3.47,2\frac{2}{9}$; therefore, the share of each, for the distance, is $\frac{1}{2}$ of $\$3.47,2\frac{2}{9} = \$1.73,6\frac{1}{9}$. For the next 52 miles, the coach was occupied by A, B, and C. The expense of the coach, for this distance, was $\$9.02,7\frac{7}{9}$, it being $\frac{52}{72+72} = \frac{52}{144}$ of \$25, of which each paid $\frac{1}{3} = \$3.00,9\frac{7}{27}$. For the next 42 miles, there were four passengers, A, B, C, and D. The expense for this distance, was $\frac{42}{144}$ of $\$25 = \$7.29,1\frac{2}{3}$, and each of the *four* passengers paid $\$1.82,2\frac{11}{12}$. For the remaining part of the distance, 30 miles, the expense was $\frac{30}{144}$ of $\$25 = \$5.20,8\frac{1}{3}$; and as E has joined them, each person bears $\frac{1}{5}$ of the expense for this distance, $\$1.04,1\frac{2}{3}$. Thus, we perceive that A and B, each, pays $\$1.73,6\frac{1}{9} + \$3.00,9\frac{7}{27} + \$1.82,2\frac{11}{12} + \$1.04,1\frac{2}{3} = \$7.60,9\frac{103}{108}$.

C pays $\$3.00,9\frac{7}{27} + \$1.82,2\frac{11}{12} + \$1.04,1\frac{2}{3} = \$5.87,3\frac{91}{108}$.

D pays $\$1.82,2\frac{11}{12} + \$1.04,1\frac{2}{3} = \$2.86,4\frac{7}{12}$; and

E pays $\$1.04,1\frac{2}{3}$.

[☞ *See the next page.*]

FORM OF STATEMENT.

$$\tfrac{20}{144}\times\tfrac{1}{2},+\tfrac{52}{144}\times\tfrac{1}{3},+\tfrac{42}{144}\times\tfrac{1}{4},+\tfrac{30}{144}\times\tfrac{1}{5}=\tfrac{526}{1728},\text{ A's product.}$$
$$\tfrac{20}{144}\times\tfrac{1}{2},+\tfrac{52}{144}\times\tfrac{1}{3},+\tfrac{42}{144}\times\tfrac{1}{4},+\tfrac{30}{144}\times\tfrac{1}{5}=\tfrac{526}{1728},\text{ B's product.}$$
$$\tfrac{52}{144}\times\tfrac{1}{3},+\tfrac{42}{144}\times\tfrac{1}{4},+\tfrac{30}{144}\times\tfrac{1}{5}=\tfrac{406}{1728},\text{ C's product.}$$
$$\tfrac{42}{144}\times\tfrac{1}{4},+\tfrac{30}{144}\times\tfrac{1}{5}=\tfrac{198}{1728},\text{ D's product.}$$
$$\tfrac{30}{144}\times\tfrac{1}{5}=\tfrac{72}{1728},\text{ E's product.}$$

$\frac{1728}{1728}$, Sum of the products.

$$\text{Then, if}\left\{\begin{array}{llllll} 1728 : & \$25 : & : 526 : & \$7.60.9\tfrac{103}{108}, & \text{A's expense.} \\ \text{"} : & \text{"} : & : \text{"} : & \text{"} & \text{B's} \quad \text{"} \\ \text{"} : & \text{"} : & : 406 : & \$5.87.3\tfrac{91}{108}, & \text{C's} \quad \text{"} \\ \text{"} : & \text{"} : & : 198 : & \$2.86.4\tfrac{7}{12}, & \text{D's} \quad \text{"} \\ \text{"} : & \text{"} : & : 72 : & \$1.04.1\tfrac{2}{3}, & \text{E's} \quad \text{"} \end{array}\right\}\text{Ans.}$$

III. BARTER.

EXAMPLES.

1. G received from H 760 lbs. of rough tallow, to try out, at 60 cts. per 100lb. clear, and was to take his pay in rough tallow, at 8 cts. per lb.; G returned 615 lbs. clear, and H paid the balance due to G in rough tallow. Allowing 18 per cent. for waste, what was the balance due to G?

* Thus, as 100—18, : 100 : : 615 : 750, then 760—750=10 lb. not tried out; then, 100 : 60 : : 615 : \$3,69, which will pay for 46⅛ lbs. rough, which, less the 10 lb. that remains in G's hands, shows the balance due to G is 36⅛ lbs., Ans.

2. A has a chest of tea, weighing 3½ cwt., the prime cost of which is £60. Now allowing interest at 6 per cent. per annum, how must he rate it per lb. to B, so that by taking his note of hand, payable at 6 months, he may clear \$50 by the bargain?

* Int.=£2 5s., and \$50=£15.

Then, 3½ cwt.÷60+15+2¼=3s. 11$\frac{8}{9}$d., Ans.

3. A merchant sold a parcel of coffee, at 15 cents per lb., and lost 10 per cent. Some time after, he sold another parcel of the same, to the amount of \$700, and gained 40 per cent. How many pounds were there in the last parcel, and at what rate was it sold?

* 100—10 : 100 : : 15 : 16⅔ cts., cost per lb.

Then, 100 : 100+40 : : 16⅔ : 23⅓ cts. per lb., rate sold at, to gain 40 per cent.; and \$700÷23⅓ cts.=3000 lbs., Ans.

4. A has linen cloth at 30 cts. per yard, ready money, in barter 36 cts. ; B has 3610 yards of ribin at 22 cts. per yard, ready money, and would have of A $200 in ready money, and the rest in linen cloth. What rate does the ribin bear in barter per yard, and how much linen must A give B ?

*If 30 : 36 : : 22 : 26 cts. 4 m., barter price of the ribin ; and $200÷22 cts.=909$\frac{1}{11}$, then 3610—909$\frac{1}{11}$=2701$\frac{10}{11}$yds., to be exchanged for linen ; then, 2701$\frac{10}{11}$×.264,÷.36=1980$\frac{3}{15}$ yards, Ans.

5. G. Jackson has 100 reams of paper at $1,33,$\frac{1}{3}$ ready money, in barter $1,66$\frac{2}{3}$; R. Howard has pamphlets at 8$\frac{1}{3}$ cts. apiece, ready money, which he adequately charges, and insists on having the money for $\frac{1}{4}$ of the price of those he parts with. What number of books should J. receive in lieu of his paper ; and how much money does H. receive, and what is his gain ?

*4 times 33$\frac{1}{3}$=$1,33$\frac{1}{3}$, which plus $\frac{1}{4}$ of itself=$1,66$\frac{2}{3}$, hence the $\frac{1}{4}$ that H. receives in money, is equal to his gain, and this $\frac{1}{4}$ is equal to $\frac{1}{3}$ of the cash price of J.'s paper ; consequently, 1.33$\frac{1}{3}$×100,÷3=$41,66$\frac{2}{3}$. H. receives.

Then, 1.33$\frac{1}{3}$×100,÷8$\frac{1}{3}$ cts.=1600 books, J. receives.

IV. COMMISSION AND BROKERAGE.

EXAMPLES.

1. My factor receives $1008 to lay out, after deducting his commission of 5 per cent. What does his commission amount to ?

*As 100 : 5 : : 1008 : $50,40, the common method, Ans.

The just method, thus, as 105 : 5 : : 1008 : $48, Ans.

The pupil will readily perceive the reason of the last being the just method, by considering that one should not pay per cent. upon the commission allowed.

2. A's commission, at 5 per cent. on a consignment of coffee was $47,50, by the gross sales of which the shipper made 25 per cent. profit. What was it invoiced at ?

*If 5 : 47,50 : : 25 : 237,50 ; then, 237,50—47,50=$190 ; then, if 25 : 237,50 : : 100 : $950, and 950—190=$760, Ans.

3. A sold goods to a certain amount, on a commission of 4 per cent., and having remitted the neat proceeds to the owner, he received $\frac{1}{4}$ per cent. for prompt payment, which amounted to $1560. What was the amount of his commission ?

*$\frac{1}{4}$: \$15,60 : : \$100 : \$6240, sum remitted ; then, if \$96 : \$100 : : \$6240 : \$6500, value of goods sold ; then, \$6500—\$6240=\$260, amount of commission, Ans.

4. If iron worth \$4 per cwt. cash, is sold for \$4,50, on a credit of 8 months, what credit should be allowed on wine worth in cash \$224 per pipe, but sold at \$242, to make the percentage equal to that on the iron ?

*If 4 : 4,5 : : 100 : 12$\frac{1}{2}$ per cent., then if 100 : 112,5 : : \$224 : \$252, and 252—224=\$28, and 242—224=\$18, then if 28 : 12$\frac{1}{2}$: : 18 : 8$\frac{1}{28}$ per cent. And, if 12$\frac{1}{2}$: 8m. : : 8$\frac{1}{28}$: 5$\frac{1}{7}$ months, Ans.

5. A, of Boston, remits to B, of New York, a bill of Exchange on London, the avails of which he wishes to be invested in goods on his account. B having disposed of the bill at 7$\frac{1}{2}$ per cent. advance, he received \$9675, and having reserved for himself $\frac{1}{4}$ per cent. on the sale of the bill, and 2 per cent. for commission ; what will remain for investment, and for how much was the bill drawn ?

*If 107,50 : : 100 : : 9675 : \$9000 : £2025 sterling, the bill ; \$100—25cts.=\$99,75 ; then, if 100 : 99,75 : : \$9675 : 9650, 81$\frac{3}{34}$; then, if 102 : 100 : \$9650,81$\frac{3}{34}$: \$9461,58$\frac{2}{4}$ for investment, Ans.

6. Bought 5 hhds. of wine at \$1 dollar per gallon, cash ; having kept it 3 months and 23 days, I sold it at \$1,20 per gallon, on a credit of 5 months ; 16 gallons having leaked out while in my possession. What was my cash gain ?

*5 hhds.=315 gallons, which cost \$315 ; and 315—16= 299 gallons saved, and 299×1,20—1,00=\$59,80 gained on what was saved ; then, 315×,06=\$18,90, int. on \$315 for one year ; then, if 360 : 18,90 : : 263 : \$13,807, which plus \$16=\$29,807, lost by int. and leakage.

Then \$59,80—\$29,807=\$29,993+gain, Ans.

V. INSURANCE.

EXAMPLES.

1. *To find the sum for which a policy should be taken out to cover a given sum.*

Q. It is required to cover \$759, premium 8 per cent. For what sum must the policy be taken ?

*Thus, as 100—8 : 100 : 759 : \$825, Ans.

2. *To find the premium per cent. when the policy is taken out for a certain sum in order to cover a given sum.*

Q. If a policy be taken out for $1250 to cover $500, what is the premium per cent ?

*Thus, as 1250 : 500 : : 100 : $40, then 100—40=$60, Ans.

3. *When the policy for covering any sum and the premium per cent. are given, to find the sum to be covered.*

Q. If a policy be taken out for $1250, at 60 per cent., what is the adventure ?

* Thus, as 100 : 100—60 : 1250 : $500, Ans.

NOTE.—The three following questions being all different, each from the other, the pupil should be informed of the principle involved in them, by the teachers' asking questions to that effect; and it might not be amiss to suggest, that the variety of principles in *Percentage Proportionals* might be easily taught in the same way; and should the pupil attend particularly to the relation that one part of a solution bears to another part, the principle of the rule to which the question belongs, will be readily discovered.

4. A adventured $500 from Boston to Baltimore at 3 per cent., from thence to Gaudaloupe at 4, from thence to Nantz at 5, and from thence home at 6 per cent. For what sum must he take out a policy to cover his adventure the voyage round?

*Thus, $\left\{ \dfrac{100\times100\times100\times100\times500}{100-3,\times100-4,\times100-5,\times100-6} \right. = \$601,278.$ Ans.

5. B adventured $480,50 from Bangor to Richmond, from thence to Jamaica, and thence home, and the premium was 5 per cent. from port to port. What amount will be equal to the several given premiums on the policy, which will cover the first adventure of $480,50 ?

*To find the policy, proceed as in the preceding solution, hence, the policy=$560,43125.

Then, $560,43125 : $480,50 : : $100 : $85,7375.

Then, 100—85,7375=$14,26¼ cts., amount required.

6. C adventured $480,50 from Belfast to Savannah, from thence to New Orleans, and thence home, to cover which all round he took out a policy for $560,43125, and the premium was equal from port to port. What was the premium per cent.

*Thus, $100-\sqrt[3]{\left\{ \dfrac{100\times100\times100\times480,5}{560,43125} \right.} = 5$ per ct., Ans.

7. D covers $200 at 6 per cent. from Portsmouth to the W. Indies, and home again; but the voyage terminating in the W. Indies, what must the insurer receive per cent?

*If 100—6 : 100 : : 200 : $212,765957, will cover the voyage round, and 100×100×200=2000000.

Then, $\left\{ \frac{2000000}{212765957} \right.$=9400, and 100—$\sqrt{9400}$=3,0465, Ans.

VI. DISCOUNT.

EXAMPLES.

1. What is the difference between the interest and discount of $350 for 8 years, at 4 per cent., simple interest?

*Interest of $350 for 8 years, at 4 per cent is $112. And 6087½ days=16⅔yrs.=200 months of 30$\frac{7}{16}$, days each, in which time, any sum of money at 6 per cent., simple interest, will double itself; hence, to find the discount of a given sum, for a given time, at a given rate per cent.—*add the given time, in months, to the number of months required by that sum to double itself at the given rate, for a divisor, to divide the product of the given sum and time, in months, and the quotient is the discount.* Hence, 200×6,÷4=300 months required for $350, at 4 per cent. to double itself.

Then, 300+8×12)350×8×12($84,848, discount of $350 at the given rate and time, then $112, int.—$84,848=$27, 152, Ans.

2. A merchant has three notes due to him as follows: One of $300, due in two months; one of $250, due in five months; and one of $180, due three months ago, with interest; the whole of which he now receives. What sum is received on the three notes, allowing money to be worth 6 per cent. a year?

*The amount of $180 for 3 months at 6 per cent. is 182,70; and the equated time for 2 months and 5 months is 3$\frac{4}{11}$months.

Then, 200+3$\frac{4}{11}$)300+250×3$\frac{4}{11}$($9,097, discount.

And, 300+250+182,70,—9,097=$723,63, Ans.

3. Sold goods to the amount of $3120, to be paid one half in 3 months, and the other half in 6 months. How much must be discounted for present payment, when money is worth 6 per cent. a year?

*The equated time for 3 and 6 months is $4\frac{1}{4}$ months.
Then, $200+4\frac{1}{2})3120\times4\frac{1}{2}($68,655256$, discount, Ans.

PRINCIPLE.—*As* $100 *will amount to* $1,06 *in one year, at* 6 *per cent. it is evident, that, if* $\frac{100}{106}$ *of any sum be taken, it will be the present worth for that time, and* $\frac{6}{106}$ *will be the discount.*

4. What is the present worth, and discount, of $600 due 3 years hence, at 6 per cent. per annum, *Compound Interest?*

*Three years time, hence 3d power of $1,06=$1,19101.
Then, 1,19101)600.000000($503.7704, present worth, } Ans.
And, 600—503,7704=$96,2296, discount. }

VII. INTEREST.

EXAMPLES.

1. If $100 in 5 years be allowed to gain $20,50, in what time will any sum of money double itself, at the same rate of interest?

*Int. of $100 for 5 years at 6 per cent., is $30, simple int.
And, if $30:6::20,5:4\frac{1}{10}$ per cent.
Then, if $6:16\frac{2}{3}::4\frac{1}{10}:24\frac{16}{41}$ years, Ans.

2. In what time will the interest of $72,60 equal that of $15,25 for 64 days, at any rate of interest?

*Int. of $15,25 for 64 days, at 8 per cent., is $0,21399, and the int. of $72,60 for 365 days, at 8 per cent., is $5,808.
Then, if $5:808:365::,21399:13\frac{5135}{11616}$ days, Ans.

3. The interest of a certain sum at simple interest for 16 years, at 5 per cent. per annum, remaining unpaid, wanted but $9,32 of the principal. What is the sum?

*$100 on interest for 16 years, at 5 per cent, is equal to $180, and 200—180=$20.
Then, if $20:100::932:$46,60, Ans.

PROBLEMS IN INTEREST.

I. *The Principal, Ratio, and Time given, to find the Int.*

RULE.—Multiply together the decimal expressing the rate per annum, the time in years and the decimal of a year, and the principal; the product will be the interest.

Note.—In the preceding rule consider $365\frac{1}{4}$ *days the year, and*

$30\frac{7}{16}$ *days the month; precision in casting simple or compound interest requiring it.*

II. *The Principal, Time, and Amount given, to find the Ratio.*

Rule.—Subtract the principal from the amount and the rem. is the int. for the time. Divide this interest by the given time, expressed in years or the decimal of a year, and the quotient will be the int. for one year. Divide the interest for one year by the given principal, and the quotient will be the rate per cent. per annum.

III. *The Principal, Ratio, and Amount given, to find the Time.*

Rule.—Divide the int., found by subtracting the principal from the amount, by the principal, and this quotient by the ratio, and the quotient is the time.

IV. *The Amount, Time and Ratio given, to find the Principal.*

Rule.—Divide the amount by the amount of one dollar for the time and rate, and the quotient will be the principal.

PARTIAL PAYMENTS.

The U. S. Court and the Courts of the several States, with the exception of Connecticut, Vermont, and New Jersey, have adopted the following rule for estimating interest on notes and bonds, when partial payments have been made:

Rule.—Cast the interest to the time when the payments shall at least be equal to the interest, then discharge the interest from the payment, subtract the excess, if any, from the principal, and cast the interest on the new principal as before, and so on.

EXAMPLE.

A had a note against B for $1166,666, dated May 1, A.D. 1796, upon which were the following payments, viz.:

1. Dec. 25, 1796, $166,666.
2. July 10, 1797, $16,666.
3. Sept. 1, 1798, $50,000.
4. June 14, 1799, $333,333.
5. April 15, 1800, $620,000.

What is due August 3, 1801?

*Set down the sum upon which the interest is to be cast, with the time, interest, and payments in columns as in the folling:

TABLE OF OPERATION.

	Principal.	Time.	Interest.	Payments.	Excess.
1	$1166,666 121,167	7m.24d.	$45,499	$166,666	$121,167
2	1045,499	6 15	33,978	16,666	
3	1045,499	13 21	71,616	50,000	
4	1045,499	9 13	49,312	333,333	
	245,093		154,906	399,999	245,093
5	800,406 579,847	10 1	40,154	620,000	579,847
	220,559	15 18	17,203	Then, 220,559 — 17,203 leaves $237,762, Ans.	

The method employed in the Courts of New Jersey, is but very little different from the preceding rule.

The rule employed in the Courts of Vermont produces results a little different from the following rule. It was established by the Supreme Court of Connecticut in 1804, *and should be studied by residents in that State. It may be omitted by others.*

CONNECTICUT RULE.

" Compute the interest on the principle to the time of the first payment; if that be one year or more from the time the interest commenced, add it to the principal, and deduct the payment from the sum total. If there be after payments made, compute the interest on the balance due, to the next payment, and then deduct the payment as above; and in like manner from one payment to another till all the payments are absorbed; *provided the time between one payment and another be one year or more.*

But if any payment be made before one years' interest hath accrued, then compute the interest on the principal sum due on the obligation for one year, add it to the principal, and compute the interest on the sum paid, from the time it was paid up to the end of the year; add it to the sum paid, and deduct that sum from the principal and interest, added as above.

And if the year extends beyond the time of settlement, find the amount of the principal remaining unpaid, up to the time of such settlement, also the amounts of the payment or payments up to the same time, and deduct their sum from the amount of the principal. If any payments be made of a less sum than the interest arisen at the time of such payment, no interest is to be computed, but only on the principal sum for any period."
Kirby's Reports, page 49.

COMPOUND INTEREST.

Rule.—The required amount of the several amounts for the several years, is the last term of an increasing series of continual proportionals, whose first term is the principal, whose ratio is the amount of one dollar for one year, and whose number of terms is the number of years plus one. *See Continual Proportionals.*

EXAMPLE.

If one mill had been put at interest at the commencement of the Christian era, what would it amount to, at compound interest, supposing the principal to have doubled itself every 12 years, Jan. 1, A.D. 1837?

*Thus, $2\times2^{153-1}=$\$11417981541647679048466287755595961091061972,99,2, Ans.

It would take the present inhabitants of our globe more than a million of years to count it; and its value in pure gold, formed into a globe, would be many millions times larger than all the bodies that compose the solar system, however incredible it may appear.

VIII. ANNUITIES.

Annuities are sums of money payable periodically, for a certain length of time, or during the life of some person, or forever.

Annuities not paid at the time they become due, are said to be in arrears.

EXAMPLES.

I. *To find the amount of an Annuity in arrears.*

1. A man hired a house for 6 years, at \$300 a year, but

did not pay the rent until the end of the term. How much did the whole amount to, allowing 6 per cent., *simple interest?*

*Thus, 300×,06×5 yrs. (No. of years less 1.)+600(twice the given sum)=690, then 690×6yrs.,÷2=$2070, Ans.

2. Suppose the salary of the Vice President of the United States, which is $5000, to remain unpaid 4 years, what would it amount to at 6 per cent., *compound interest?*

*Thus, 1.06×1.06×1.06×5000=$21873,08, Ans.

3. If a salary of $120 payable quarterly, remain unpaid 7 years, what would it amount to in that time at 6 per cent. per annum, *simple interest?*

*For quarterly payments, take ¼ of the ratio, ¼ of the annuity, and 4 times the number of years; for half yearly payments, take ½ the ratio, ½ the annuity, and double the years.

¼ of 6 per cent. is $1,50, ¼ of $120 is $30, and 4 times the number of years is 28 yrs., the sum of the natural series of which, less 1, is 94,6⅓; and the interest of $30 for 1 year is $1,80; then, 1,8×94,6⅓=$170,33⅓, equal to the whole interest due; then, $30×28 yrs.=840, and 840+170,33⅓=$1010,33⅓, Ans.

4. What is due on a pension of $150 a year, payable half-yearly, but forborne 2 years; allowing half yearly *compound interest*, at 4½ per cent. per annum?

*½ of 4½ per cent.=,0225, which plus unity,=amount of one dollar of next to last payment, ½ of the annuity=$75, and 75×1,0225=$76,6875, the amount of next to the last payment.

Then 1,0225×1,0225=1,04550625, and 75×1,04550625 =$78,41296875, the amount of the second payment.

Then 1,04550625×1,0225=1,069030140625, which multiplied by 75=$80,17726034675, the amount of the first payment, and since the last payment of an annuity does not draw any interest, the sum of these amounts, with the last payment, is $310,277729096875, Ans.

II. *The Annuity, Amount and Ratio given, to find the Time.*

5. In what time will $60 per annum, payable yearly, amount to $262,47696, allowing 6 per cent. compound interest?

*Thus, 262,47696×1,06,+60=338,2255776, from which subtract 262,47696=75,7486176,÷60=1,26247696, amount of one dollar for the required time; then, 1,26247696,÷1,06,

$\div$1,06,$\div$1,06, and the Rem. again by 1,06=0, which shows four divisions by 1,06, hence 4 yrs., Ans.

III. *The Amount, Ratio and Time given, to find the Annuity.*

6. What annuity being forborne 4 years, will amount to \$262,47696, at 6 per cent. compound interest?

*Thus, $\overline{1,06}|^4$—1=,26247696, *the divisor.*

Then, 262,47696×1,06,—262,47696=15,7486176, which divided by the *divisor*, gives \$60, Ans.

IV. *The Annuity, Ratio and Time given, to find the Present worth.*

7. What is the present worth of the lease of a house for 3 years, the rent being \$54,85 a year, and the ground rent being 2½ guineas a year, the rent payable half yearly, discount being allowed at 10 per cent. compound interest?

*Thus, 2½ guineas=\$7,35, which taken from \$54,85 shows \$47,5 clear, ½ of which is \$23,75, and 3 yrs. doubled is 6 yrs., and ½ of 10 per cent. is 5 per cent., and the amount of \$1 annuity for 6 yrs. is \$6,801913, and the amount of \$1 for the same rate and time, is \$1,340096.

Then, 6,801913×23,75,$\div$1,340096=\$120,65625, Ans.

Note—I have given the examples at simple interest in annuities, more for the gratification of the curious than for real utility, it being not only customary, but also most equitable, to allow compound interest.

V. *The Present worth, Time and Ratio given, to find the Annuity.*

*Multiply the Tabular number, corresponding with the rate and time, by the purchase money; the product will be the annuity.

VI. *The Annuity, Present worth and Ratio given, to find the Time.*

8. For how long may an annuity of \$60 per annum be purchased for \$207,906336762, at 6 per cent. compound interest?

*Thus, 60+207,906336762,—207,906336762×1,06)60(1,26247696; then, 1,26247696$\div$1,06,$\div$1,06,$\div$1,06,$\div$1,06=0, shows 4 divisions by 1,06, hence 4 years, Ans.

VII. *The Annuity, Time and Ratio given, to find the Present worth of the Annuity in Reversion.*

9. What is the present worth of \$60 payable yearly, for 4 years; but not to commence until two years hence, at 6 per cent.?

*Present worth of \$1 for the time in being and reversion is equal to 4,91732.

Again, by the Table of Present worth of the time in being, \$1 is 1,83339. Then 4,91732—1,83339=3,08393, and 3,08393×60=\$185,0358+, Ans.

The reverse of the preceding operation:

$$\text{Thus, } \frac{185{,}0358\times1{,}26247\times1{,}1236\times1{,}06-1}{1{,}26247-1}=\$60,\ \text{Ans.}$$

and is practised when it is required to find the annuity, when the present worth of the reversion, rate and time are given.

VIII. *The Ratio and Annuities which are to continue on unlimited time being given, to find the Present worth of Reversion.*

10. What is the value of a rent of \$60 per annum, to commence 2 years hence, allowing the purchaser 6 per cent.?

*Thus, ,06)60,00(\$1000, its present worth.

Then, 1,06×1,06)1000(\$889,9966, required, Ans.

The reverse of the preceding operation: Thus, price of the reversion, $889{,}9966\times\overline{1{,}06}|^2\times{,}06=\$60$, Ans., and is practised when it is required to find the annuity, when the value of a reversion, the time prior to its commencement, and rate are given.

IX. *Annuities which are to continue an unlimited time and the Ratio being given, to find the Present worth.*

11. What is the present worth of \$60 per annum, allowing 6 per cent. to the purchaser?

*Thus, as 6 : 100 : : 60 : \$1000, Ans.

X. *The Present worth of an Annuity for an unlimited time and the Ratio being given, to find the Annuity.*

*As 100 is to the rate, so is the present worth to the annuity; and when the present worth and yearly rent are given, to find the rate, say, as the present worth is to the rent, so is 100 to the rate.

XI. *When the Ratio is given, to find the number of years' purchase an Annuity may be bought.*

*Divide 100 by the rate, and the quotient will be the years; and by dividing 100 by the number of years that you would buy an annuity you will have the rate.

XII. ***An Annuity, several times in reversion, and rate being given, to find the several present values.***

12. A has a term of 6 years in an estate of \$60 per annum. B has a term of 14 years in the same estate, in reversion, after the expiration of the 6 years; and C has a further term of 16 years, after the 20 years are expired. I demand the present value of the several terms, at 6 per cent. compound interest.

*Pres. value of \$1 for years 36=14,61722,×60=877,0332.
" " " \$1 " 6+14=11,46992,×60=688,1952.
" " " \$1 " 6= 4,91732,×60=295,0392
dollars, the present worth of A's term.

Then, 688,1952—295,0392=\$393,156, pres. value of B's.
And, 877,0332—688,1952=\$188,838, " " " C's.

13. Suppose I would add 5 years to a running lease of 15 years, the rent being \$186,375; what ought I to pay down for the favor, discount being allowed at 5 per cent., compound interest?

*Amount of \$1 dollar annuity for 5 years, at 5 per cent. is \$5,525631, and the amount of \$1 for 20 yrs. at 5 per cent. is \$2,653298.

Then, 5,525631×186,375,÷2,653298=\$388,135+, Ans.

14. Which is the most advantageous, a term of 15 years in an estate of \$100 per annum, or the reversion of said estate forever after the said 15 years, computing interest at 5 per cent. per annum, compound interest?

*Present worth of \$1 annuity for 15 years=\$10,379658, ×100=\$1037,9658, the present worth of \$100 annuity for 15 years; then \$100÷5cts.=\$2000, the value of the estate; and the amount of \$1 for 15 yrs. at 5 per cent. is \$2,078928; then, 2000÷2,078928=\$962,0342, the present worth of the estate in reversion.

Then, \$1037,9658—962,0342=\$75,9316 that the present worth of the annuity is worth more than the present worth of the whole estate.

I. EVOLUTION.

Evolution is the reverse of Involution; for in Involution we have the root given, to find the power; but in Evolution we have the power given to find the root.

Power and root are correlative terms; for, as 4 is the square of 2, 2 is the square root of 4, and the same of any power or Root.

The extraction of the root is finding a number which being multiplied into itself the requisite number of times, will reproduce the given number; for example, if we extract the square root of 64, we find it to be 8, and 8×8 is equal to 64, &c.

Hence the root is designated by the number of times it is used as factor in producing the corresponding power. It is used twice in producing the second power and is called the second or square root, &c.

Any number which is the exact root of any power, is a rational number, and its power a perfect number; and since any number may be the root of its corresponding power, it follows that any root which can be exactly expressed by figures, is a rational number. But there are numbers whose roots can never be exactly extracted, and these numbers are called imperfect powers, and their roots are called irrational numbers, or *surds*. For example, 2 is not only an imperfect power of the second degree, but an imperfect power of any degree, and not only its square root, but the root in every degree is irrational, or a surd; because no number, either whole or fractional, can be found, which, being involved to any degree, will produce 2. The same is true of many other numbers.

Some numbers are perfect powers of one degree, and imperfect powers of another degree.

Surds occur when we endeavor to find a root of any number, which is not a perfect corresponding power.

II. SQUARE ROOT.

The terms *square* and *square root*, are derived from geometry, which teaches us that the area of a square is found by multiplying one of its series by itself.

AREA signifies the space included on any geometrical figure.

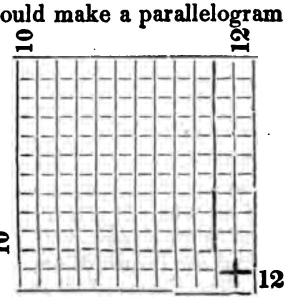
DEMONSTRATION.

The points placed over one of the figures in each period, to prepare the number for extracting its root, is to direct what figures to bring down, without the trouble of counting them.

The first point is placed over the unit figure, instead of the left hand figure, because, if the pointing should begin at the left hand, the true root would not always be obtained, and the point would sometimes fall on the place of *tenths*, in decimals, when there were decimals in the number. But when the num-

ber of integral figures is *odd*, and the *square* root is wanted, or *even*, and the *cube* root wanted, it is immaterial whether you begin to point at the right or left hand, as the effect will be the same in both cases. Any number is distinguished into periods of as many figures as the index of the root denotes, because the second power can never have more than twice as many figures as its root, and never but one less than twice as many. The third power can never have more than three times as many figures as its root, and never but two less than three times as many. The same holds true of the higher powers and their roots. For instance, take two numbers consisting of any number of places; but let them be the least possible of those places, viz.: Unity with ciphers, as 100, and 10. Then their product will be 1 with so many ciphers annexed as are in both the numbers, viz.: 1000; but 1000 has 1 place less than 100, and 10 together; and since 100, and 10 were taken the least possible, the product of any other two numbers, of the same number of places, will be greater than 1000. Q. E. D. *(quod erat demonstrandum.)**

The root is doubled for a divisor, for if it was not doubled, the quotient or next part of the root would be all on one side of the square before found, which would make a parallelogram instead of a square. For example, suppose the figure in the margin is now 12 squares in length and 12 squares in breadth, each square being an inch on either side but was, before it was ascertained by the following operation that it would make a foot square, a strip of surface just one inch wide and 144 inches in length.



Operation by the common method.

$$
\begin{array}{r|l}
 & 144(12 \\
 & 1 \\
 \hline
 22) & 44 \\
 & 44 \\
 \hline
 & \cdot\cdot
\end{array}
$$

Operation as understood.

$$
\begin{array}{r|l}
 & 144(10+2 \\
 & 100 \\
 \hline
 20) & 44 \\
 & 40 \\
 \hline
 & 4 \\
 & 4 \\
 \hline
 & \cdot
\end{array}
$$

* Q. E. D. signifies, *which was to be demonstrated.*

EXPLANATION.

By the common method of operation, the first figure of the root, 1, is in reality 10, as may be seen by the second operation, because the square of 10 is 100. The root 1, of 1, or 10, of 100, doubled for a divisor, is 2 in appearance but 20 *as understood*, and 2 is contained twice in 4, and 20 is contained twice in 44, and 4 remains.

The second figure of the root, 2, is a piece 2 inches wide, and 10 inches in length, making 20 square inches to add to one side and one end of the first or ten inch square, both pieces containing 40 square inches, which, with the square of the last figure of the root, to fill the vacant corner, completes the square, whose side is 12, which appears plain from the preceding figure and by the operation *as understood*.

After finding *one more than half* the required number of figures in the root, the rest may be found and the operation greatly abridged by dividing the last remainder by the last divisor, which, with the quotient figure last annexed, is used as a constant divisor, and the division is continued by omitting, continually, the last figure of the divisor on the right hand, at each division.

III. CUBE ROOT.

A CUBE is a solid body, having six equal sides, which are all squares, the length, breadth and depth being equal.

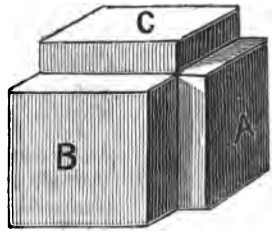
The *cube root* of a number is a number which, being multiplied by its square, will produce the *given* number. Or, it is to find the length of one side of a solid body the contents of which is expressed by the given number.

DEMONSTRATION.

In extracting the cube root, you divide by 300 times the square of the quotient, because when the root consists of two figures, and that first found is 1, it is 10 units, and 300 times the square of 1 is the same as 3 times the square of 10, and all other numbers are governed by the same principle.

The reason of 3 times the square of 10, when there are but two figures in the root and the first of which is 1, being concerned in the extraction of the cube root, is because, the cube being a solid body, contained under six equal square sides, if an

addition be made to it so as to equally increase the length of each side, it must be done by adding three equal squares, one on each of two of its sides, and one on the top or bottom of the first cube, (See each piece of the figure,) each square being of equal superfices as the square of one side of the cube to which it is attached; and when the side of the cube is 10, the square of the side is 100, and as 100 must be added to *three* sides, it will require 300 for them all. After the three squares are added to the first cube, there are three vacancies to fill, the end of each being a square whose side is the last quotient figure, therefore, the product of the last quotient figure squared, multiplied by all the others, shows how many times the square of the last figure is contained in the tenth part of the length of one of the three vacancies, each of which is equal in length to the side of the first cube.



This last product is multiplied by 30, because if the side of the cube is 10, there being three vacant corners of equal length, the sum of all three is 30, and the product of 30 by the square of the last quotient figure multiplied by all the others, supplies the deficiency of the three long vacant spaces, (See preceding figure,) but since the second addition comes no higher up nor further out than the first addition, the cube of the last figure is added to complete the diagram. *Hence the reason of the following*

RULE.

First, let the numbers pointed be
In periods each of figures three;
The cube of your left period take
And of its root a quotient make;

Which root into a cube must grow
And from the period taken fro;—
To the remainder then you must
Bring down another period just;

Which being done then you must see
This number right divided be
By just three hundred times the square
Of what your quotient figure are.

Multiply the divisor by
The last quotient figure ; apply
Last figure squared, perhaps the *last*
When multiplied by all the rest,

And then increased just thirty times,
(Which fills the vacant corner lines ;)
Last figure cubed and added too,
Would be put in, if right you'd do :

All this would be *the subtrahend;*
Thus ply each period to the end.
If any thing remain, you shall
Add triple ciphers for decimals.

NOTE.—It is presumed, the pupil will take more interest in the extraction of the cube root, since the rule is in the form as here given, hence the reason for its being so given.

IV. A NEW METHOD OF EXTRACTING ROOTS.

For simplicity, brevity, and accuracy, it is undoubtedly the best method now known. Its superiority over the old method, will be readily seen, as it saves more than half the labor. Teachers most certainly should be acquainted with it.

EXAMPLE.

Required, the cube root of 122615327232.

OPERATION.

		122615327232(4968, *root.*
4 . . .	16	64
8 . . .	48	58615, first dividend.
129 . .	5961	53649, *the subtrahend.*
138 . .	7203	4966327, second dividend.
1476 . .	729156	4374936, *the subtrahend.*
1482 . .	738048	591391232, third dividend.
14888 .	73923904 . . .	591391232, ...

NOTE.—In this operation there are 113 figures. The same example performed by the old method, would require at

least 252 figures; and to leave the example so that the process could be traced, it would require 318 figures.

EXPLANATION.

The numbers being pointed off as usual, I find the root of the greatest cube in the left-hand period to be 4. This is placed in the quotient, and also at the head of the first or left-hand column; its square, 16, is placed at the head of the second column, and its cube, 64, is placed under the left-hand period, and subtracted therefrom. To the remainder, I bring down the next period, making 58615, the first dividend. The 4, at the head of the first column, is doubled and tripled, (because we are extracting the *third* root,) and set underneath, making 8 and 12. The product of 8 by the quotient figure, 4, is added to 16, making 48, the first divisor. I then seek how many times the divisor, 48, is contained in the dividend, 58615, (excepting the two right-hand figures,) that is, in 586, and find it is contained 9 times; placing 9 in the quotient, and also at the right-hand of 12, making 129. The 9 is then doubled and tripled, and set underneath, making 138 and 147. Then I multiply 129 by the 9, and add the product to 48, taking care to remove the product two places to the right-hand of 48. (See 5961 under 48 in the second column.) This, multiplied by 9, gives 53649, the subtrahend, which being subtracted from the first dividend, and the next period, 327, being brought down, makes 4966327, the second dividend. Then I multiply 138 by 9, and add the product to 5961, making 7203, the second divisor, which is contained in the dividend, 4966327, (excepting the two right-hand figures,) 6 times. To find the second subtrahend, or the third and last divisor, proceed as for either of the preceding.

The last figure of the root, 8, is placed in the quotient, and also at the right-hand of 1488, (See the operation) making 14888, which is multiplied by 8, and the product added to 738048 advancing the product two places to the right; the sum is 73923904. This, multiplied by 8, and placed under the dividend, finishes the operation.

Note.—It is to be remembered, that, in dividing, the number of figures omitted at the right-hand of the dividend, must always be one less than the index of the given power; that is, if we are extracting the third or cube root, we must omit *two*

figures at the right-hand of the dividend; if the 5th root, we must omit *four*, &c. The *operation* can be all carried on mentally, by this method, without the necessity of making a figure any where else.

When the divisor is larger than the dividend, a cipher is placed in the root, and one at the right-hand of the number in the first column opposite the next subtrahend, and the next period brought down, and one more figure taken, &c.

The 4th root is most conveniently performed, by two extractions of the square root, instead of the new method.

Double the root is the Den. to a Rem. in square root; and triple the square of the root, plus triple the root is the Den. to a Rem. in finding the cube root of a given number.

The preceding rule answers practical purposes *only*, but it is the method for finding the true Den. to the numbers contained between any square or cube number, and their next succeeding square or cube.

PERMUTATIONS AND COMBINATIONS.

I. PERMUTATIONS.

PERMUTATION means the different ways in which the order or relative position of any given number of things may be changed.

Numbers in Permutation have no two arrangements in which all the quantities have the same relative position.

CASE I.—*To find the number of changes that can be made of any given nnmber of things, all different from each other.*

Thus—how many changes may be rung on seven bells?
* $1\times2\times3\times4\times5\times6\times7=5040$, Ans.

CASE II.—*To find the number of changes that may be made in the arrangement of a given number of things, in which there are several things of one sort, several of another, &c.*

Thus—how many changes can be made in the order of the letters a a a b b c?

* If the letters in this question were all different, they would admit of $1\times2\times3\times4\times5\times6=720$ variations; but since a is found 3 times, we must divide that number of variations by $1\times2\times3$; and, since b occurs twice, we must again divide by 1×2; hence, the number of variations will be $\frac{1\times2\times3\times4\times5\times6}{1\times2\times3\times1\times2}=60$, Ans.

Case III.—*Any number of different things being given, to find how many changes can be made out of them, by taking a given number at a time.*

Rule.—Take a series of numbers commencing with the given number of things, and decreasing by 1, till the number of terms is equal to the number of things to be taken at a time, and the product of all the terms of this series will be the answer.

EXAMPLE.

How many changes can be rung with 4 bells out of 8?

* $8\times7\times6\times5=1680$, Ans.

II. COMBINATIONS.

Combination consists in taking a less number of things out of a greater, without any regard to the order in which they stand.

No two combinations can have the same quantities.

Case 1.—*To find the number of combinations from any given number of things, all different from each other, taking a given number at a time.*

Rule I.—Take the series 1, 2, 3, &c., up to the number of things to be taken at a time, and find the product of all the terms.

2. Take a series of as many terms, decreasing by 1, from the given number, out of which the election or choice is to be made, and find the product of all the terms.

3. Divide the last product by the first, the quotient is the number sought.

EXAMPLE.

How many combinations can be made of 6 letters out of the 24 letters in the Alphabet?

$$* \ \frac{24\times23\times22\times21\times20\times19}{2\times3\times4\times5\times6}=134596, \text{ Ans.}$$

CASE II.—*To find the various combinations of a given number of things, which may be made out of an equal number of sets of different things, one from each set.*

RULE.—Multiply the number of things in the several sets continually together, and the product will be the answer.

EXAMPLE.

Suppose there are 4 companies, in one of which there are 6 men, in another 8, and in each of the other two nine men. What are the choices, by a composition of 4 men, one out of each company?

* $6\times8\times9\times9=3888$ choices, Ans.

PROGRESSION,

ARITHMETICAL AND GEOMETRICAL.

1. PROGRESSION ARITHMETICAL,

OR, EQUIDIFFERENT SERIES.

When the numbers increase, they form an ascending series; but when they decrease, a descending series. Thus, 1, 2, 3, 4, 5, 6, 7, 8, 9, form an ascending series, because they continually increase by 1; but 9, 8, 7, 6, 5, 4, 3, 2, 1, form a descending series, because they continually decrease by 1.

The numbers which form the series, are called the *terms* of the series. The first and last terms in the series, are called the *extremes;* and the other terms the *means.* The number by which the terms of the series are continually increased or diminished, is called the *common difference.*

PRINCIPLES ASSUMED.

1. When four numbers form a progressional series, the sum of the two extremes is equal to the sum of the two means; and of any *three* quantities, in such a series, double the mean is equal to the sum of the extremes.

2. In any equidifferent series, the sum of the two extremes is equal to the sum of any two means that are equally distant from the extremes, and equal to double the middle term, when there is an uneven number of terms.

3. The difference between the extremes of an equidifferent series, is equal to the common difference multiplied by the number of terms less 1.

4. The sum of all the terms in any equidifferent series, is equal to the sum of the extremes, multiplied by half the number of terms.

The reason of these Principles, and the following Problems or cases, it is presumed, is sufficiently obvious without being demonstrated.

For brevity and perspicuity, let a=first term, l=last term, n=number of terms, d=common difference, s=sum of all the terms, and let the different cases be represented as in the following Table. By this means, a summary of the whole doctrine of equidifferent series may be presented at a single view.

In Progression, if three parts be given, the other two may readily be found. By an explanation of the first problem or case in the Table, the rest will be readily understood. In the first case of the Table, read—*The first term, last term, and number of terms given, to find the common difference; or, sum of all the terms.*

RULE.—Divide the difference of the extremes by the number of terms less 1, the quotient will be the difference. Multiply the sum of the extremes by the number of terms, half the product will be the sum of all the terms.

A SYNOPSIS OF THE DIFFERENT CASES IN EQUIDIFFERENT SERIES.

CASE	GIVEN.	REQUIRED.	RULES, OR METHOD OF SOLUTION.	CASE	GIVEN.	REQUIRED.	RULE, OR METHOD OF SOLUTION.
1	aln	d	$\frac{l-a}{n-1}$	6	ans	d	$\frac{2,\times s-a\times n}{n-1,\times n}$
		s	$\frac{a+l\times n}{2}$			l	$\frac{2s}{n}-a$
2	ald	n	$\frac{l-a}{d}+1$	7	lds	a	$\frac{d+-,\sqrt{2l+d^2-8ds}}{2}$ $(+-,=+ \textit{or} -)$
		s	$\frac{l+a,\times l-a+d}{2d}$ (2d=twice d)			n	$\frac{2l+d+-,\sqrt{2l+d^2-8ds}}{2d}$
3	als	d	$\frac{l+a,\times l-a}{2s,-l+a}$ (2s=twice s)	8	lns	a	$\frac{2s}{n}-l$
		n	$\frac{2s}{a+l}$			d	$\frac{2,\times n\times l-s}{n-1,\times n}$
4	ads	n	$\frac{\sqrt{\overline{2a-d}\rceil^2+8ds},-2a-d}{2d}$ (8ds=8d×s)	9	lnd	a	$l,-n-1\times d$
		l	$\frac{\sqrt{\overline{2a-d}\rceil^2+8ds},-d}{2}$			s	$n,\times l-n-1\times\frac{d}{2}$
5	adn	l	$n-1,\times d+a$	10	dns	a	$\frac{s}{n}-\frac{d,\times n-1}{2}$
		s	$n,\times a+\overline{n-1}\times\frac{d}{2}$			l	$\frac{s}{n}+\frac{d,\times n-1}{2}$

II. PROGRESSION GEOMETRICAL,

OR, CONTINUAL PROPORTIONALS.

Any series of numbers, the terms of which gradually increase or decrease by a constant multiplication or division, are said to be *Continual Proportionals.*

The number by which the series is constantly increased or diminished, is called the ratio.

The first and last terms of a series are called *extremes,* and the other terms *means.*

PRINCIPLES ASSUMED.

1. If four quantities be in continual proportion, the product of the two means will be equal to that of the two extremes, when continued, or if discontinued; and, of three quantities, the square of the mean is equal to the product of the two extremes.

2. If four quantities are such, that the product of two of them is equal to the product of the other two, then are those quantities proportional.

3. If four quantities are proportional, the rectangle of the means, divided by either extreme, will give the other extreme.

4. The products of the corresponding terms in continual proportions, are also proportional.

5. If three numbers be in continued proportion, the square of the first will be to *that* of the second, as the first number to the third.

6. In any continual proportion, the product of the two extremes, and *that* of every other two terms, equally distant from them, are equal.

7. The sum of any number of quantities, in continued proportion, is equal to the difference of the rectangle of the second and last terms, and the square of the first, divided by the difference of the first and second terms.

¶ As the *last term,* or any term near the last, is very tedious to be found by continual multiplication, it will be very necessary, in order to ascertain it, to have a series of numbers in arithmetical proportion, called *indices,* or *exponents,* beginning either with a cipher, or a unit, whose common difference is one.

When the *first* term of the series and the *ratio* are *equal,* the indices must begin with a unit; and, in this case, the pro-

duct of any two terms is equal to that term signified by the *sum* of their indices.

Thus, 1, 2, 3, 4, 5, 6, &c., indices, or arithmetical series.
And 6+6=12, *index*.

Then 2, 4, 8, 16, 32, 64, &c., geometrical series (leading terms) of the twelfth term.

And 64×64=4096, *the twelfth term.*

But, when the *first* term of the series and the *ratio* are *different*, the *indices* must begin with a cipher, and the sum of the *indices*, made choice of, must be *one less* than the *number of terms*, given in the question; because 1 in the *indices* stands over the *second term*, and 2, in the *indices*, stands over the *third term*, &c.; and, in this case, the *product* of any *two* terms, divided by the *first*, is equal to that term *beyond* the first, signified by the *sum* of their indices.

Thus, 0, 1, 2, 3, 4, 5, 6, &c.; indices.
And, 6+5, *index* of the 12th term.
Then, 1, 3, 9, 27, 81, 243, 729, &c., geometrical series.
And, 729+243, *the twelfth term.*

8. If the *ratio* of any geometrical series be *double*, the *difference* of the *greatest* and *least* terms is equal to the sum of all the terms, except the greatest; if the *ratio* be *tripled*, the *difference* is *double* the sum of all the terms, except the greatest, &c., &c.

9. In any geometrical series decreasing, and continued *ad infinitum*, *half* the *greatest* term is equal to the *sum* of all the remaining terms, *ad infinitum*. *Let a=first or least term, l=last or greatest term, s=sum of all the terms, n=number of terms, r=ratio, L=logarithm, and the different cases be represented as in the following Table.* By this means, a summary of the whole doctrine of Continual Proportionals may be presented at a single view.

By an explanation of the first problem or case in the following Table, it is presumed the different problems will be easily understood. In the first case of the Table, read—*The first term, ratio, and number of terms given, to find the sum of all the terms.*

RULE.—The ratio, less 1, raised to the power denoted by the number of terms, and divided by the ratio less 1, the result multiplied by the first term.

A SYNOPSIS OF THE DIFFERENT CASES IN CONTINUAL PROPORTIONALS.

CASE	GIVEN.	REQUIRED.	RULE, OR METHOD OF SOLUTION.
1	arn	l	$a \overset{+}{\div} r^{n-1}$ (*exponent.*) ($\overset{+}{\div} = +$ *or* $\div$, as may be)
		s	$\frac{r^{n}-1}{r-1} \times a$ (n=index of r—1.
2	arl	s	$l + \frac{l-a}{r-1}$
		n	$\frac{L.\ l - L.\ a}{L.\ r} + 1$ ($l \times a = L.l - L.a$)
3	ars	l	$\frac{r-1, \times s + a}{r}$
		n	$\frac{L.\ r-1, \times s + a - L.\ a}{L.\ r}$
4	als	r	$\frac{s-a}{s-l}$
		n	$\frac{L.\ l - L.\ a\ (\text{or } l \times a)}{L.\ s-a, - L.\ s-l} + 1$
5	ans	r	$\frac{rs}{a} - r^{n-1} = \frac{s-a}{a}$ (n—1, see case 1)
		l	$\overline{l \times s - l\vert}^{\,n-1} = a \times \overline{s-a\vert}^{\,n-1}$

REMAINDER OF CASES IN CONTINUAL PROPORTIONALS.

CASE.	GIVEN.	REQUIRED.	RULE, OR METHOD OF SOLUTION.
6	anl	r	$\sqrt[n-1]{\frac{l}{a}}$ ($\sqrt{}$ = reverse of $\lvert^{m}$
		s	$l+\frac{l-a}{\sqrt[n-1]{\frac{l}{a}}-1}$ { $\sqrt[n-1]{}$ = index of $l\div a$
7	rnl	a	$\frac{l}{r^{n-1}}$
		s	$l-\frac{l}{\overline{n-1}}$ ($\overline{n-1}$ = index of r) $l+\frac{r}{r-1}$
8	rns	a	$\frac{r-1}{r^{n}-1}\times s$ (n = index of r—1)
		l	$\frac{r^{n}-r^{n-1}}{r^{n}-1}\times s$
9	rls	a	$s-r\times\overline{s-l}$
		n	$\frac{L.\,l-L.\,\overline{s-r\times\overline{s-l}}}{L.\,r}$
10	nls	a	$a,\times\overline{s-a}\rvert^{n-1}=l\times\overline{s-l}\rvert^{n-1}$
		r	$r^{n}+\frac{s}{l-s}\,r^{n-1}=\frac{l}{l-s}$

PHILOSOPHICAL PROBLEMS.

GRAVITY—SPECIFIC GRAVITY—MECHANICAL POWERS—FALLING BODIES—RIVERS AND FLUIDS—PENDULUMS—AIR-BALLOON—BAROMETER—SUPPLEMENT.

I. GRAVITY OR WEIGHT.

PRINCIPLES.

1. The gravity of any body above the earth's surface *decreases*, as the squares of its distance in semi-diameters of the earth from its centre *increase*.

2. If the diameters of two globes be equal, and their densities different, the weight of a body on their surfaces will be as their densities.

3. If their densities be equal, and their diameters different, the weight of a body on their surfaces will be in proportion to their diameters.

4. If the diameters and densities be both different, the weight of a body on their surfaces will be as the product of their diameters and densities.

5. As the sum of the square roots of the quantities of matter in two globes or planets, is to the distance of their centres, so is the square root of the quantity of matter in either, to the distance from its centre at which a body would be suspended between both by the attraction of each in a contrary direction.

PROBLEMS, RULES, AND EXAMPLES.

PROB. I.—Knowing the weight of a body at the earth's surface, to know what it will weigh at any height above the earth.

RULE.—This is contained in Principle I., which see; also, see the following

EXAMPLE.

A certain body on the surface of the earth, weighs 400lbs. What will it weigh at 2000 miles above the surface?

*2000+4000=1,5 semi-diameter from the earth's centre, hence, 1,5×1,5, square of the semi-diameter, by which divide the weight of the body, 400lbs., and the quotient, 178 lbs. is the Ans.

Prob. II.—Given, the weight of a body at any distance above the earth's surface, to know what it will weigh at the surface.

Rule.—This is also contained in Principal I. It is the reverse of Prob. I.; and proves the preceding operation.

EXAMPLE.

If a body weighs 178lbs. at 2000 miles above the earth's surface, what will it weigh at the surface?

*$2000+4000=1,5$, semi-diameter, then $178\times1,5\times1,5=$ 400lbs., Ans.

Prob. III.—Knowing the weight of a body at the earth's surface, to find how high it must be carried to be of any other weight.

Rule.—Divide the weight at the surface by the weight in the air above the surface, and the square root of the quotient will be the answer in semi-diameters of the earth from its centre.

EXAMPLE.

If a body weighs 320lbs. at the earth's surface, how high must it be carried to weigh but 20lbs.?

*$\sqrt{320\div20}=4$ semi-diameters of the earth from its centre, Ans.

Prob. IV.—To find how high a body must be raised above the earth's surface, to retain any given proportion of its weight.

Rule.—See Principle I.; also, see the operation of the following

EXAMPLE.

How high must a ball be raised to lose half its weight?

As $1:\overline{4000}|^2::2:32000000$, the square root of which, less 4000, is 1656,85 miles, Ans.

Prob. V.—Knowing the weight of a body at the earth's surface, to find how much it will weigh at the surfaces of the other planets.

Rule.—See Principle 4, and the operation of the following

EXAMPLE.

If a stone weighs 100lbs. at the surface of the earth, what

*In relation to all these problems, the earth's diameter is considered to be 8000 miles; though some estimate it at 7964,12, others at 7928, or 7911,73 miles.

will it weigh at the moon's surface, if its density and diameter is as may be found in the following table. The upper number is the density, and the lower one the diameter.

Sun.	Jupiter.	Saturn.	Earth.	Moon.
$1\frac{2}{15}$	$1\frac{1}{24}$	$0\frac{13}{32}$	$4\frac{1}{2}$	$3\frac{2}{7}$
883246.	89170.	79042.	7911.73.	2180.

Mercury.	Venus.	Mars.	Herschel.
$9\frac{1}{6}$	$5\frac{1}{15}$	2	$\frac{99}{100}$
3224.	7687.	4189.	35112.

*As $7911.73 \times 4.5 : 100 :: 2180 \times 3\frac{2}{7} :$ to the number of pounds required.

NOTE.—The order of the planets from the Sun, according to their distances, is Mercury, Venus, Earth and Moon, Mars, Jupiter, Saturn, and Herschel.

PROB. VI.—To find at what distance from any two planets a small body would be suspended between them both, the distance of the two planets being known.

RULE.—This is contained in Principal 5.

EXAMPLE.

At what distance from the earth would a body be suspended between the earth and Moon?

*If the quantity of matter in the earth be to that of the Moon as 1 to ,025; and the distance of their centres, 240000 plus $3955.86+1090=245045.86$.

Then, as $\sqrt{1}+\sqrt{.025} : 245055.86 :: \sqrt{1} :$ a number of miles from which subtract 3955.86, and the remainder will be the answer in miles.

A problem and rule is contained in the following

EXAMPLE AND OPERATION.

How high will the attraction of the earth raise a tide on the moon, if the moon raise a tide on the earth 5 feet high, and supposing there are seas or oceans in the moon.

*As $\overline{2180}|^3 \times 3\frac{2}{7} : 5 :: \overline{7911.73}|^3 \times 4\frac{1}{2} :$ a fourth number *directly*.

And, as 2180 : *the 4th No.* :: 7911.73 : to the required height in feet *inversely*. And the reason of such is this, the attraction of the moon on the earth, or of the earth on the moon, is directly as its quantity of matter, and inversely as its diameter.

II. SPECIFIC GRAVITY.

DEFINITION AND PRINCIPLE.

1. The specific gravity of a body is its weight compared with the weight of its bulk of pure water, a cubic foot of water weighing 1000 ounces and being 1.000.

2. If a body which will sink in water be first weighed in air and then in water, it will lose so much of what it weighed in air, as its bulk of water.

REMARKS. 1.—If a body, whose specific gravity is required, is lighter than water, affix to it another body heavier than water, so that the mass, compounded of the two, may sink together, and to solve the questions of this nature, see the solution of the second example.

2. In obtaining the specific gravities of gold and silver; instead of the specific gravities, you may obtain the same result by substituting the weight of a cubic inch, or of any other part of the compound, and the weight of the same bulk of gold and silver.

3. The specific gravity of liquids, is found by weighing a solid body in *air*, in *water*, and in *another* fluid, and to solve questions relating to fluids, see the Problem and examples that relate to fluids.

PROBLEMS, RULES AND EXAMPLES.

PROB. I.—*To find the Specific Gravity of Bodies.*

RULE.—Divide the weight in air by the loss in water, and the quotient will be the specific gravity.*

EXAMPLES.

1. If a piece of gold weighs 516 grains in air, and loses 29 grains by weighing it in water, what is the specific gravity of the gold?

*Thus, 516÷29=17,793 times its bulk of pure water, Ans.

2. If a piece of elm weighs 15lbs. in air, and a piece of copper weighing 18lbs. in air, and 16lbs in water is affixed to it, and the compound weighs 6lbs in water. Required the specific gravity of the elm.

*Divide the weight by the bulk, the quotient is the specific gravity; and, divide the weight by the specific gravity, the quotient is the magnitude.

*Thus, 18lbs.—16lbs.=2lbs.; 15lbs.+18lbs.=33lbs.; 33lbs., less 6lbs.+2lbs.=25lbs.

Then, as 25lb. : 15lb. : : 1000oz. : 600oz., or $\frac{3}{5}$ as heavy as its bulk of water, Ans.

PROB. II.—*To discover the quantity of alloy in metals.*

RULE.—If the specific gravity of the compound be lighter than some of the articles in the mixture, subtract the specific gravity of the compound from the specific gravity of such articles; and subtract the specific gravity of the lighter articles of the mixture from the specific gravity of the compound. The first remainders will show the proportional bulk of the heavier articles of the mixture, and the latter that of the lighter articles. Multiply the differences by their respective specific gravities, and the products will be the proportional *weights* of each metal in the compound. Then as the sum of the proportional weights is to the whole weight of the compound, so is each proportional weight to the real weight of its kind in the whole compound or mixture.

EXAMPLE.

If a crown of gold and silver weigh 60 ounces, and the specific gravity of the compound is 15, what part of the crown is gold, and what part is silver?

*In "*Table of Specific Gravities*," 18,888 is the specific gravity of Standard Gold, and 10,535 that of Standard Silver.

$$\text{Then, } 15 \begin{cases} 18{,}888 & 4{,}465, \text{ proportional bulk of gold.} \\ 10{,}535 & 3{,}888, \quad \text{“} \quad \text{“} \quad \text{“ silver.} \end{cases}$$

And, $18{,}888 \times 4{,}465$, proportional weight of gold, and
$10{,}535 \times 3{,}888$, " " " silver.

Then find the quantity of each article of the mixture by the last clause of the preceding rule.

PROB. III.—*To find the Specific Gravity of fluids or liquids.*

RULE.—From the weight of a solid body, when weighed in air, take its weight in water, and the remainder is the weight of its bulk of water. Also, from its weight in air, take its weight in the fluid concerned, and the remainder is the weight of the same bulk of that fluid. Divide the weight last found by the weight of the same bulk of water, and the quotient is the specific gravity of the fluid or liquid of which you would find the relative weight.

EXAMPLES.

1. A cubic inch of common glass weighs about 1,36oz. Troy; a cubic inch of salt water ,5427oz.; a cubic inch of brandy ,48927oz. Suppose then, B has a gallon of brandy in a bottle, which weighs 4½lbs. Troy, out of water, and to conceal it, throws it into the sea. Will it sink or swim, and by how much is it heavier or lighter than the same bulk of salt water?

*4½ lbs.÷1,36oz.=39,7059,+231=270,7059, cubic, inches in the bottle and brandy.

Then, 270,7059×,5427=146,912oz., weight of salt water occupied by the bottle and brandy.

And ,48927×231,+54 is the weight of the bottle and brandy, which is 20,11oz. heavier than its bulk of salt water, hence the bottle and contents will sink.

2. What is the specific gravity of French brandy, consisting of 5 parts, measure, of rectified spirits of wine and 3 parts water?

*850×5,+1000×3=7250,÷5+3=,906,25, specific gravity.

III. MECHANICAL POWERS.

LEVER—WHEEL AND AXLE—INCLINED PLANE—WEDGE—PULLEY—AND SCREW.

DEFINITIONS AND PRINCIPLES.

A LEVER is any inflexible bar, which serves to raise weights, if it is supported at a point, which is the centre of its motion, by a *fulcrum* or prop.

1. PRINCIPLE. To balance a large weight by a small one; the distance between the fulcrum and the end of the lever to which the power is applied, must be as much greater than the distance between the fulcrum and the weight to be raised, as the weight to be raised is greater than that which is to raise it.

THE WHEEL AND AXLE is a wheel turning round together with its axis; the power is applied to the circumference of the wheel, and the weight to that of the axis by means of cords.

2. PRINCIPLE. The power must be to the weight as the radius of the axle is to the radius of the wheel.

An inclined plane is a plane which makes an acute angle with the horizon.

3. Principle. A power must be employed, parallel to the plane, which shall be to the weight of the body as the height of the plane is to its length.

The wedge is two inclined planes, the base of each being joined to the other.

4. Principle. The power must be to the resistance, as the thickness of the head is to the length of both of the slanting sides.

The pulley is a small wheel, movable about its axis by means of a cord, which passes over it.

5. Principle. When the axis of a pulley is fixed, the pulley only changes the direction of the power. If movable pulleys are used, an equilibrium is produced, when the power is to the weight, as one to the number of ropes applied to them. If each movable pulley has its own rope, each pulley will double the power.

The screw is a spiral thread or groove, cut round a cylinder, and every where making the same angle with the length of the cylinder. In one round of the spiral, it rises along the cylinder the distance between two threads.

6. Principle. The power must be to the weight as the distance between the tops of the coils is to the circumference described by the lever.

Remarks.—*Weight and Power*, when opposed to each other, signify *the body to be moved and the body that moves it.*

In operating with the screw, wedge or pulley, one third of the power is lost in overcoming friction, and some friction is to be overcome in operating with the wheel and axle, or the inclined plane.

PROBLEMS, RULES AND EXAMPLES.

I. THE LEVER.

Prob. I.—*Given, the power, the fulcrum's place, and the length of the lever, to find the weight that may be raised.*

Rule.—This is contained in Principal 1; hence, when any three parts be given, by a little consideration, the other may be readily found; also, see the following

EXAMPLES.

1. If a man weighing 150lbs. rest on the end of a lever 10ft. long, what weight will he balance on the other end, if the prop or fulcrum be 2ft. from the weight?

*As 2ft. : 8ft. (10ft.—2ft.) : : 150lbs. : 600lbs., Ans.

2. In giving directions for making a chaise, the length of the shafts, between the axletree and back-band, being settled at 9ft., a dispute arose whereabout on the shafts the centre of the body should be fixed. A advised to place it 30 inches, B 20 inches before the axletree. If the body of the chaise and its burthen are 420lbs., what will the beast in both cases bear more than its harness?

*9ft.=108 inches ; then, 108 : 420 : : $\left\{\begin{matrix} 30 : 116\frac{2}{3}\text{lbs.} \\ 20 : 77\frac{7}{9}\text{lbs.} \end{matrix}\right\}$ Ans.

3. A cheese in one scale weighed 76lbs., in the other scale 56 lbs. Required its true weight.

*The square root of the product of the different weights is the true weight.

Thus, $\sqrt{76\times 56}=65\frac{31}{130}$lbs., Ans. The den., 130=double the root, 65.

4. A cheese in one scale weighed 90lbs., in the other scale 40lbs. Required its true weight, and the proportional lengths of the two arms of the balance beam on each side of the fulcrum.

*Thus, $\sqrt{90\times 40}=60$lbs. *its true weight;* then, 90—60=30, and 60—40=20. Then, if 30lbs. : 3ft. : : 20lbs. : 2ft. ; hence the arms of the scales are to each other as 2ft. to 3ft.

5. Two men carrying a burthen of 200lbs., hung on a pole, the ends of which rest on their shoulders; how much of this load is borne by each man, the weight hanging 6 inches from the middle, and the whole length of the pole being 6 feet?

*4ft.=48in. ; 6in.×2=12 in. ; then, 48—12=36in. ; and, 36÷2=18in. ; then, 18+12=30.

Then, as 48 : 200 : : 18 : 75 lbs., Ans.

And, as 48 : 200 : : 30 : 125lbs., Ans.

II. THE WHEEL AND AXLE.

PROB. II.—*Given, three parts of the wheel and axle, to find the fourth or required part.*

RULE.—See Principal 2, and the following

EXAMPLES.

1. I would make a windlass in such a
of power should equal 10lbs. of weigh
axle, it being 6 inches in diameter. Req
the wheel.

*Inversely, as 10 : 6 : : 1 : 60 inches, A

By a little consideration, when any th
parts are given, the other may be readily

2. There are two wheels; one of whic
with an axle of 9 inches diameter; and the
ameter, with an axle of 7 inches diameter.
Suppose the power cord of the smaller
upon the axle of the larger; what weigh
smaller wheel would be balanced by 100lb.
of the larger ?

*First, as 9in. : 6ft. : : 100lb. : 800lb., t
the smaller wheel; then, as 7in. : 4ft. : : 80

III. THE INCLINED PLAN

PROB. III.—*To find the power that will d
inclined plane.*

RULE.—Multiply the weight by the perpe
the plane, and divide this product by the len

EXAMPLES.

1. A certain inclined plane is 16 feet i
in perpendicular height; what weight might
plane by a power, which, if exerted on a cord
pulley, would raise 25lbs. ?

*As the height, 7 : 16, length : : 25, power

Also, 16 : 27 : : 7 : 25 lbs., power.

N. B.—When any three of the preceding
questions relating to this mechanical power,
found by attending to Principle 3.

2. If a set of pulleys, 3 of which are m
to a weight upon an inclined plane of 5
14 feet perpendicular height; what weigh
would be sustained by 40lbs. at the power c

*Thus, as 14 : 50 : : 40×3×2 : 857lbs.

IV. THE WEDGE.

REMARK.—The wedge is of the same nature as the inclined plane, calling the breadth of the head of the wedge, the perpendicular height of the plane, and the length of its side, the length of the plane, and the force acting against the head of the wedge, the power.

EXAMPLE.

If a wedge be 12 inches long, and its head $1\frac{1}{2}$ inch broad; and if a screw whose threads are $\frac{3}{4}$ of an inch asunder, be applied to the head of this wedge; and if a power of 200lbs. were applied to the end of the lever, 16 feet long; what would be the force exerted on the sides of the wedge?

*,75in. : 3,141592×16×2×12 : : 200 : 3216990,208.

15in. : 12in. : : 3216990,208lbs. : 2573592,116lbs, Ans. (See Principles 4 and 6.)

V. THE PULLEY.

RULE.—As unity is to twice the number of movable pulleys, so is the power to the weight. Reverse the statement to find the power.

When any three parts of this mechanical power are given, to find a required part, see Principle 5.

EXAMPLES.

1. If a cord which runs over 3 movable pulleys, be attached to an axle 4 inches in diameter, the wheel of the axle being 38 inches in diameter, and a power of 20lbs. be exerted at the circumference of the wheel; what weight would be raised under the pulleys?

*Thus, as 4in. : 38in. : : 20lbs.×3×2 : 1140lbs., Ans.

2. There is a power equal to 30lbs. and a weight of 200lbs.; if one end of a rope is fastened to a block of fixed pulleys, how many movable pulleys must there be so that the power shall raise the weight?

*As 30lbs.—$\frac{1}{3}$ of 30 lbs., allowed for friction, is to unity, so is 200lbs. to twice the number of movable pulleys, Ans. 5.

VI. THE SCREW.

REMARK.—When any three properties of the screw are given to find a required part, proceed by Principle 6.

IV. FALLING BODIES.

PRINCIPLES.

1. Heavy solid bodies near the surface of the earth, fall about 16 feet in the first second, that is, one foot in the first quarter, 3 feet in the second quarter, &c., increasing each last quarter, for every next quarter, 2 feet, in *common* air; but, in *vacuo*, falling bodies acquire to 16,1 feet in the second.

2. The space in feet through which a body has fallen, is equal to the square of the time in which it was falling, in fourths of a second.

3. The velocities acquired by falling bodies, in a different number of seconds, are in proportion to the squares of their times; hence, the number of feet a body falls in any second, beginning at one, may be found by multiplying 16 feet by the odd numbers, 1, 3, 5, &c.

Thus, in 5 seconds; $16\times1=16$; $16\times3=48$.
And, $16\times5=80$; $16\times7=112$; $16\times9=144$, &c.

4. Ascending bodies are retarded in the same ratio that descending bodies are accelerated.

PROBLEMS, RULES AND EXAMPLES.

PROB. I.—*The velocity of a body moving in any direction, being given, to find how far a body must fall to acquire the same velocity.*

RULE.—Divide the velocity by 8, and the square of the quotient will be the space fallen through.

Reason.—Because the square root of the space fallen through, is always equal to one-eighth of the velocity acquired at the end of the fall.

EXAMPLE.

If the velocity of a cannon ball is 660 feet per second, from what height must a body fall to acquire the same velocity?

* $660\div8=82{,}25$, then $82{,}25\times82{,}25=6806\frac{1}{4}$ feet, Ans.

PROB. II.—*The time in which a body has been falling, being given, to find the distance or space fallen through.*

RULE.—Multiply the time in seconds by 4, and the square of the product will be the distance required.

Reason.—Because the square root of the feet fallen through, is always equal to four times the number of seconds the body has been falling.

EXAMPLES.

1. What is the difference between the depth of two wells, into each of which, should a stone be dropped at the same instant, one would reach the bottom in 5 seconds, and the other in 3 seconds?

* $5\times4=20$, and $20\times20=400$; $3\times4=12$; $12\times12=144$. Then, $400-144=256$ feet, Ans.

2. A ball was seen to fall half the way from the top of a tower in the last second of time. How long was it in descending, and what was its height before its descent?

* The $\sqrt{1}=1$, $\sqrt{2}=1,4142$; and $1,4142-1=,4142$.

Then, as $.4142:1.4142::1:3.414+$ seconds, time of its descent.

Then, $3.414\times3.414\times4=186.424+$ feet, the space fallen through.

PROB. III.—*Given, the velocity per second, to find the time a body has been falling.*

RULE.—Divide the velocity by 8, and a fourth of the quotient is the time.

Reason.—Because four times the number of seconds in which a body has been falling, is equal to one-eighth of the velocity, in feet, per second, acquired at the end of the fall.

EXAMPLE.

How long must a bullet be falling to acquire the velocity of 160 feet per second?

* $160\div8=20$, and $20\div4=5$ seconds, Ans.

PROB. IV.—*Given, the number of feet fallen through, to find the time of a body's falling.*

RULE.—Divide the square root of the distance fallen through by 4, and the quotient will be the answer in seconds.

EXAMPLE.

In what time will a ball dropped from the top of a steeple 144 feet high, reach the ground ?

*Thus, $\sqrt{144}=12$, and $12\div4=3$ seconds, Ans.

PROB. V.—*To find the velocity per second, with which a heavy body will begin to fall, from any distance above the earth's surface.*

RULE.—As the square of the earth's semi-diameter, in miles, is to 16 feet, so is the square of any other distance, in miles from the earths's centre, inversely, to the velocity required.

Reason.—See Principle 1, ("Gravity or Weight,") and Principle 1, ("Falling Bodies.")

EXAMPLE.

With what velocity will a heavy ball begin to descend per second, if raised 1000 miles above the earth's surface ?

*As $4000\times4000:16::5000\times5000:10{,}25$ feet, Ans.

PROB. VI.—*To find the velocity per second acquired by a falling body, or by a stream of water, at the end of any given time, in seconds, the perpendicular descent or fall being given.*

RULE.—Multiply the space fallen through by 64, and the square root of the product will be the answer.

Reason.—Because the velocity acquired at the end of any number of seconds, is equal to twice the mean velocity with which the body falls during that time.

NOTE.—If a question requires the reverse of a statement as by the rule, it will be when the velocity with which a falling body strikes an obstacle, is given, to find the perpendicular space fallen through.

EXAMPLE.

1. There is a short flume, one end of which is $2\frac{1}{2}$ feet higher than the other. What would be the velocity of a stream of water through it per second ?

*$2{,}5\times64=160$, and $\sqrt{160}=12{,}649$ feet, Ans.

The reverse, thus, $12{,}649\times12{,}649,\div64=2\frac{1}{2}$ feet, Ans.

NOTE.—If the velocity per second, 12,649, be multiplied by 62,5 (lbs. per cubic foot,) for clear water, by 63 lbs. for dirty water, and by 64 lbs. for sea water, the product would be the

momentum or force, a fluid running with such velocity, would strike against a fixed obstacle.

Prob. VII.—*Knowing the weight of a body, and the space fallen through, to find the force with which it will strike.*

Rule.—Find the velocity by Problem VI., and the product of the weight and velocity will be the answer.

Note.—If the weight and striking force is given, to find the space fallen through.

Rule.—Divide the force by the weight, the quotient is the velocity. Then divide the square of the velocity by 64, and the quotient is the space fallen through.

EXAMPLE.

1. If the hammer used in driving the piles of Charlestown Bridge, weighed $2\frac{1}{4}$ tons, and fell through a space of 10 feet; with what force did it strike the pile?

*$2\frac{1}{4}$ tons$=$4500lbs., and $\sqrt{10\times64}=25,3$, velocity; then, $4500\times25,3=113850$lbs., Ans.

The reverse; thus, 113850lbs., force,$\div4500=25,3$, and $\overline{25,3}|^2\div64=10$ feet, space, Ans.

Prob. VIII.—*Given, the number of seconds from the time a body projected directly upwards returns to the earth, to find how high it ascended.*

Rule.—Multiply half the time, in seconds, by 4, and the square of the product is the answer.

Reason.—See Principle 4.

EXAMPLE.

If a ball discharged from a gun, directly upwards, returns to the earth in 14 seconds, how high did it ascend?

*$14\div2=7$, and $7\times4=28$; then, $28\times28=784$ feet, Ans.

Prob. IX.—*To find the depth of a well by dropping a stone into it, also the time of the stone's descent and of the sound's ascent.*

Rule.—Take a line of any length, and by Prob. II. *("Pendulums,")* find the time from the dropping of the stone until you hear it strike the bottom. Then, multiply this time by 73088 and to this product add 1304164, and from the square root of the sum take 1142. Divide the square of the remainder by 64, the quotient will be the depth of the well, in feet.

2. Divide the depth by 1142, the quotient will be the time of the sound's ascent, which, being taken from the whole time, will be the time of the stone's descent, in seconds.

Reason.—$73088=16\times4\times1142$; 1142 feet being the distance, which sound moves in a second; and $1304164=1142$ squared; and, $64=16\times4$; for the reason of 16×4, see Principle 2.

EXAMPLE.

I dropped a stone into a well, and a string with a plummet, which measured 25 inches from one end of the string to the middle of the plummet, made 5 vibrations before I heard the stone strike the bottom. Required the depth of the well, time of the stone's descent, and of the sound's ascent.

*$\sqrt{25\div39.2}=.7985, \times5=4$ seconds, from the dropping of the stone to the hearing of it strike the bottom.

Then, $\sqrt{73088\times4, +1304164}, -1142=121.53$.

And $121.53, \times121.53, \div64=230.77$ feet, depth of the well.

Then, $230.77\div1142=.2$ of a second, time of the sound's ascent.

And, $4-.2=3.8$ seconds, time of the stone's descent.

V. RIVERS AND FLUIDS.

PRINCIPLES.

1. If the breadth of a river is uniform, the motion of the water is accelerated in the same manner with any body moving down an inclined plane. And the force with which a body descends down an inclined plane is to that with which it would descend freely in air, as the elevation of the plane is to its length, or as the sine of the angle of inclination is to radius.

2. The water at the bottom of a river runs faster than the water at the surface. And the velocity of each drop of water in a river of uniform depth, is as the square root of its distance from the level of the surface of the fountain, and that distance is greater to the bottom than to the surface of the river.

3. The depth of a river continually decreases as it runs, if not fed by tributary streams. Because, the farther a river runs,

the swifter it runs and the more water passes in the same time, leaving less water to follow.

PROBLEMS, RULES AND EXAMPLES.

PROB. I.—*Given, the height of a head of water above the sluice, to find what the height must be to discharge any given proportion more.*

RULE.—To the square root of the first height, add the given proportion of the root, and the square of the sum will be the answer.

EXAMPLE.

If a head of water be $6\frac{1}{4}$ feet above the sluice, how high must it be to discharge one fifth as much more in the same time?

*$\sqrt{6.25}=2.5$, which plus a fifth of itself is 3.

Then, $3\times3=9$ feet, height, Ans.

N. B.—For other problems, their rules and examples, see problem 6, its Rule, Reason, Notes, and Example, ("FALLING BODIES.")

NOTE.—Water being a yielding substance, it loses two-thirds of its power in producing effects.

P. S. PROB. II.—*To find the perpendicular pressure of fluids on the bottom of vessels.*

RULE.—Multiply the area of the bottom by the altitude of the fluid, and that product by its specific gravity, gives the pressure required.

EXAMPLE.

Suppose a vessel is 3 feet wide, 5 feet long, and 4 feet high. What is the pressure on the bottom, it being filled with water to the brim?

*$3\times5=15$ square feet, area of the bottom.

Then, $15\times4=60$ cubic feet.

Then, $60\times62.5=3750$lbs.$=1$ ton 13cwt. 1qr. 26lbs. Ans.

PROB. III.—*To find the quantity of pressure against the sluice or bank, which pens a body of water.*

RULE.—Multiply the area of the sluice, under water, by the depth of the centre of gravity, (which is equal to half the depth of water,) in feet, and that product again by $62\frac{1}{2}$ for clear water, or 64 for sea-water, the product is the answer, in pounds.

EXAMPLE.

If the length of a sluice be 30 feet, and the depth of the water 4 feet. What is the pressure against the side of the sluice?

*$30 \times 4 = 120$ feet, area of the side, and 120×2, (depth of the centre of gravity,) gives 240 cubic feet.

Then, $240 \times 62.5 = 15000$lbs., Ans.

VI. PENDULUMS.

PRINCIPLES.

1. The time of an oscillation, in the cycloid, is to the time of a heavy body's descent through half its length, as the circumference of a circle to its diameter, which is as 3.1416 to 1; and a heavy body descends freely, by the attraction of gravitation, 193.5 inches in the first second, nearly, in our latitude.

2. The time of the vibration of a pendulum vibrating in a chord of a circle, is equal to the time in which a body falling freely, would descend through eight times the length of the pendulum.

3. The times of the vibrations of pendulums of different lengths, are as the square roots of their lengths.

4. The vibrations of a pendulum of the same length, are all performed in nearly the same time, whether the vibrations are large or small, but as the time of the latter is rather the least, a great number of them together would make a sensible difference of time, and as in cold weather the wheels of a clock are obstructed considerably more than in warm, so as not to communicate so much force to the pendulum, a clock will gain time in winter; another cause of its gaining is, the cold contracts the length of the pendulum rod a little, which makes it vibrate rather faster.

5. The power of gravity is greatest at the poles of the earth, and least at the equator, hence, the greater the latitude of a place, the longer must be the pendulum, to vibrate in any given time. A pendulum vibrating seconds, in lat. 51° 31′, is $\frac{1}{10}$ of an inch longer than one vibrating seconds, at the equator.

N. B.—39.2 inches is the length of a pendulum vibrating seconds in lat. 51° 31′.

PROBLEMS, RULES AND EXAMPLES.

PROB. I.—*To find the length of a pendulum vibrating in any given time.*

RULE.—Multiply the square of the time in seconds, by 39.2, and the product will be the length of the pendulum in inches.

EXAMPLE.

Required the lengths of several pendulums, which will respectively swing 4th seconds, half seconds, seconds, and minutes.

*$.25 \times .25 \times 39.2 = 2.45$ inches, to swing 4th seconds.

$.5 \times .5 \times 39.2 = 9.8$ inches, to swing half seconds.

$1 \times 1 \times 39.2 = 39.2$ inches; or, thus, as

$3.1416 \times 3.1416 : 1 :: 193.5 : 19.6$ inches, half the length, Ans. 39.2 in. for sec.; $60 \times 60 \times 39.2 = 2$ miles and 1200 feet, to swing minutes.

PROB. II.—*To find the time which a pendulum of any given length will swing.*

RULE.—Divide the given length by 39.2 and the quotient will be the square of the time in seconds.

EXAMPLE.

How often will a pendulum of 9.8 inches vibrate in a second?

*Thus, $\sqrt{9.8 \div 39.2} = .5$ of a second, that is, it vibrates half seconds.

VII. AIR BALLOON.

PROBLEMS, RULES AND EXAMPLES.

PROB. I.—*Given, the weight to be raised by a balloon, to find its diameter.*

RULE.—As the specific difference between common and inflammable air is to one cubic foot, so is any weight to be raised, to the cubic feet contained in the balloon. Divide the cubic feet by .5236, and the cube root of the quotient will be the diameter required; but, to raise it, the diameter must be something greater, or the weight something less.

EXAMPLE.

I would construct a spherical balloon, of sufficient capacity to ascend with 4 persons, weighing one with another, 160 lbs., and the balloon and a bag of sand weighing 60 lbs. Required, the diameter of the balloon.

* By the "Table of Specific Gravities," I find a cubic foot of common air weighs 1.25 oz., and a cubic foot of inflammable air, .12 oz., each Avoirdupois Weight.

Then, $1.25-.12=1.13$ oz., difference.

And, $160\times40,+60=700$ lbs.$=11200$ oz.

Then, as 1.13 : 1 ft. : : 11200 oz. : 9911.5044, the cube root of which is 26.65 feet, required diameter, Ans.

PROB. II.—*Given, the diameter of a balloon, to find the weight it is capable of raising.*

RULE.—Multiply the cube of the diameter by .5236, and the product will be the content in cubic feet. Then as a cubic foot is to the specific difference between common and inflammable air, so is the content of the balloon to the weight it will raise.

EXAMPLE.

The diameter of a balloon is 26.65 feet. What weight is it capable of raising?

* Thus, $26.65\times26.65\times26.65\times.5236=9911.5044$ cubic feet.

Then, as 1 cubic foot : 1.13 : : 9911.5044 : ounces in 700 lbs., nearly, Ans.

VIII. THE BAROMETER.

THE BAROMETER is so formed, that a column of quick-silver is supported within it, to such a height as to counterbalance the weight of a column of air, of an equal diameter, extending from the barometer to the top of the atmosphere.

At the surface of the earth, the height of this column of quick-silver is, at an average, almost 30 inches.

Cold contracts this column of quick-silver, and heat expands it.

Hence, the formation of this instrument for measuring heights.

EXAMPLES, ETC.

1. What is the pressure of atmosphere on a square foot, and on the surface of a man's body, estimated at 14 square feet?

* By the "Table of Specific Gravities," a cubic foot of quick-silver is 13600 oz., Avoirdupois; the height of the barometer is 2.5 feet; then, $13600 \times 2.5 = 2125$ lbs., on a square foot; hence, 29750 lbs. on a man's body.

2. If the mercury in a barometer, at the bottom of a tower, be observed to stand at 30 inches, and, on being carried to the top of it, be observed at 29.9 inches. What is the height of the tower?

* The specific gravity of air, 1.25)13600(10880, tenths, equal 1088 in.$=90\frac{2}{3}$ ft., Ans.

REMARK.—The number of feet, in height, of the atmosphere, corresponding with $\frac{1}{10}$ of an inch on the barometer, is variable, depending upon the temperature and density of the atmosphere.

This variation, depending on the temperature, may be shown by the Thermometer, (see "Sundry Tables,") which is calculated for every 5 degrees, from 32 to 80, Farenheit; the intermediate degrees may be easily calculated, by allowing .21 of a foot for each degree.

The altitude, thus found, will be to the altitude corrected for the density of the air, inversely, as the mean height of the barometer, at the two stations, is to 30 inches; hence the following

RULE.—Multiply the mean height corresponding to the mean temperature of the barometer at both stations, (found in the Table,) by the *tenths* of an inch in the difference of the barometer at both stations, and this product by 30; divide this last product by the mean height of the barometer at both stations, the quotient will be the answer, or height required, with the error of a few feet only, if the height be less than a mile.

EXAMPLE.

In measuring a height, at the first station, suppose the barometer to stand at 29, and the thermometer at 40; at the second station, the barometer at 28, and the thermometer at 40. What is the height of the second station above the first?

OPERATION.

Add $\left\{\begin{array}{l}\text{1st station}=29,\\ \text{2d station}=28,\end{array}\right\}\div 2=28.5$, mean height of the barometer, at the two stations; $29-28=1=10$ tenths of an inch.

Then, $\left\{\begin{array}{l}\text{1st station}=60,\\ \text{2d station}=40,\end{array}\right\}\div 2=50$, thermometer's mean height, against which, in the Table, you will find 90.66, mean temperature of the barometer at both stations. Then, by the Rule, $90.66\times10\times30,\div28.5=954.3$ feet, height of the second station above the first.

NOTE.—In obtaining the mean height of the thermometer, or barometer, for more than two stations at one measurement, we must add half of the two extremes to the sum of the means, and divide this sum by the number of intervals.

IX. SUPPLEMENT.

OF BODIES PUT IN MOTION.

PRINCIPLES.

1. If the quantity of matter, or weights of any two bodies, put in motion, be equal, the force by which they are moved, will be in proportion to their velocities.

2. If the velocities of these bodies be equal, their forces will be directly as the quantities of matter contained in them, that is, as their weights.

3. If both the quantities of matter and the velocities be unequal, the forces, with which the bodies are moved, will be in a proportion compounded of their quantities of matter and velocities.

SUNDRY EXAMPLES.

1. Suppose the battering ram of Vespasian weighed 60000 lbs.; that it was moved at the rate of 24 feet in a second, and that this was sufficient to demolish the walls of Jerusalem. With what velocity must a cannon ball, which weighs 42 lbs., be moved, to do the same execution?

* Thus, if 60000 : 24 : : 42 : $34285\frac{5}{7}$ feet in a second, Ans.

2. A body, weighing 30 lbs., is impelled by such a force as to send it 20 rods in a second. With what velocity would a body weighing 12 lbs. move, if it was impelled by the same force?

* Thus, if 12 : 20 : : 30 : 50 rods per second, Ans.

3. If the earth's mean distance from the sun be 95 millions of miles, at what distance from him must another body be placed, that it may receive a degree of light and heat double to that of the earth?

* Thus, $\sqrt{95000000 \times 95000000, \div 2} = 67175144+$ miles, Ans.

4. A is sitting 3, and B 6 feet distant from a fire. How much hotter is it at A's than at B's seat?

* Inversely, as, $6 \times 6 : 1 : : 3 \times 3 : 4$, hence A's place is 4 times as hot as B's, Ans.

N.B.—Effects produced by beds of attracting substances are in proportion to the squares of the distance. This principle extends to the two preceding questions.

5. A lens of 47 inches diameter, and 38 inches focal distance, will condense the solar rays to 17.257 times a degree of heat, and melts copper ore in 8 seconds; and, the power of such glasses for burning, are as the area of the lens, directly, is to the square of the focus' distance, inversely; hence, if a lens of the preceding dimension produce the preceding effect, in what time would copper ore be melted by a lens of 36 inches diameter, and 80 inches focus?

* First, $\left.\begin{array}{l} 47 \times 47 \times 11, \div 14 : 38 \times 38 : : 17.257 \\ 36 \times 36 \times 11, \div 14 : 80 \times 80 \end{array}\right\} : 44.873+$ times a degree of heat.

Then, if 17.257 : 8 : : 44.873 : 3.07+ seconds, Ans.

MENSURATION.

HEIGHTS, DISTANCES, SUPERFICIES AND SOLIDS.

DEFINITIONS.

REMARK.—The *definitions* that may be found in a Dictionary, containing the principal words of the English language, are omitted.

1. A *point* is that which has position, but no magnitude, nor dimensions; neither length, breadth, nor thickness.

2. A *line* is length without breadth, or thickness.

3. A *straight* or *right line* is the shortest way from one point to another. When a line is mentioned simply, it means a right line.

4. A *chord* of a circle is a right line joining the extremities of an arch, and is the measure of the arch.

NOTE.—The chord of an arc of 60 degrees, is equal in length to the radius of the circle of which the arc is a part.

5. The *sine* of an arc is a line drawn from one end of the arc, perpendicular to the radius or diameter drawn through the other end; or, it is half the chord of double the arc.

NOTE.—The sines on the same diameter, increase in length till becoming radius, decreasing after. Hence, radius is the greatest possible sine.

6. The *versed sine* of an arc, is that part of the diameter or radius which is between the sine and circumference.

7. The *tangent* of an arc is a right line touching the circumference, and drawn perpendicular to the diameter; and is terminated by a line drawn from the centre, through the other end of the arc.

NOTE.—The tangent of an arc of 45 degrees, is equal in length to the radius of the circle of which the arc is a part.

8. The *secant* of an arc, is a line drawn from the centre, through one end of the arc, till it meets the tangent.

9. The *complement* of an arc, is what it wants of 90 degrees.

10. The *supplement* of an arc, is what the arc wants of 180 degrees.

NOTE.—The sine of any arc is the same as that of its sup-

plement. So, likewise, the tangent and secant of any arc are used also for its supplement. Also, the sine, tangent, or secant of the complement of any arc, is called the co-sine, co-tangent, or co-secant of the arc. And the sine, tangent, or secant of an arc, is also the sine, tangent, or secant of the angle whose measure the arc is.

11. Angles are right or oblique, acute or obtuse.

12. If a right line meet a right line, and incline to neither side, making the angles on each side equal, then those angles are called *right angles*.

13. Every angle less than a right angle, is an *acute angle;* and every angle greater than a right angle, is an *obtuse angle.* An *oblique* angle is either obtuse or acute.

14. A *plane* is a surface, in which any two points being taken, the straight line joining those points lies wholly within that surface.

NOTE.—Plane figures, that are bounded by right lines, have names according to the number of their sides, or of their angles; for they have as many sides as angles, the least number being three, which is called a triangle, and receive particular denominations from the relations of its sides and angles.

15. *Equal figures* are those, which being applied, the one to the other, coincide entirely; thus, two circles having the same radius, are equal, &c.

NOTE.—Equal figures are always similar, but similar figures may be very unequal.

16. In two different circles, *similar arcs, similar sectors, similar segments,* are such as correspond to equal angles at the centre.

17. A *solid angle* is the angular space comprehended between several planes which meet in the same point.

18. An *equilateral triangle* is that whose three sides are equal.

19. An *isosceles triangle* is that which has two sides equal.

20. A *scalene triangle* is that whose three sides are unequal.

21. A *right-angled triangle* is that which has one right angle.

22. An *oblique-angled triangle* may be either acute or obtuse.

23. An *obtuse-angled triangle* has one obtuse angle.

24. An *acute-angled triangle* has its three angles acute.

25. A *spherical triangle* is a part of the surface of a sphere comprehended by three arcs of great circles.

NOTE.—A spherical triangle takes the name of *right-angled, isosceles* and *equilateral,* like a plane triangle, and under the same circumstances.

26. A *segment* is any part of a circle bounded by an arc, and its chord. The smaller part is understood when the contrary is not expressed.

27. A *spherical segment* is the portion of a sphere comprehended between two parallel planes, which are its basis. One of these planes may be a tangent to the sphere, in which case the spherical segment has only one base.

28. A *sector* of a circle is a space contained between two radii, and an arc less than 180 degrees, or a semi-circle.

29. A *spherical polygon* is a part of the surface of a sphere terminated by several arcs of great circles.

30. Every section of a sphere made by a plane, is a circle; hence, a *great circle* is the section made by a plane which passes through the centre, and a *small* circle is the section made by a plane which does not pass through the centre.

31. A *circle,* mathematically described, is a surface which has length and breadth, and *circumference* is only a line.

32. A *rhomboid* or *rhomboides* is a figure bounded by four sides, the opposite ones being equal, but the angles oblique.

33. A *trapezoid* is a figure bounded by four sides, two of which are parallel, though of unequal lengths.

34. A *trapezium* is a figure bounded by four unequal sides.

35. A *spherical pyramid* is the part of a sphere comprehended between the planes of a solid angle, whose vertex is at the centre. The *base* of the pyramid is the spherical polygon intercepted by these planes.

36. The *sector of a sphere* is composed of a segment less than a hemisphere, and a cone, having the same base with the segment, and its vertex in the centre of the sphere.

37. A *paraboloid,* or *parabolic conoid,* is a solid formed by the revolution of a parabola about its axis.

38. A *spheroid* is a solid generated by the revolution of an ellipse, or oval, about the transverse or conjugate diameter.

39. Every solid terminated by planes, is called a *solid polyedron*, or, simply, a *polyedron*.

40. A *cylinderoid* is a solid similar to a cone; one base may be an ellipsis, and the other a disproportional ellipsis or circle.

41. A *prismoid* is a solid similar to the frustum of a pyramid, but its bases are disproportional.

42. The *axis of a solid*, is a line drawn from the middle of one end to the middle of the opposite end; as between the opposite ends of a prism.

43. The *height*, or *altitude* of a solid, is a line drawn from its vertex, or top, perpendicular to its base.

NOTE.—As we have not been able to obtain engravings to represent any of the preceding figures, the Teacher will, I trust, have the goodness to draw them for the Pupil, on paper, the slate, or on the black-board.

It is very necessary to thoroughly understand the peculiar properties of the preceding named figures; therefore, intense application is necessary.

PRINCIPLES ASSUMED.

The following Propositions are Demonstrably true:—

1. Two triangles are equal when two sides and the included angle of the one are equal to two sides and the included angle of the other, each to each.

2. In an isosceles triangle, the angles opposite to the equal sides are equal.

3. Of the two sides of a triangle, that is the greater which is opposite to the greater angle; and, conversely, of the two angles of a triangle, that is the greater which is opposite to the greater side.

4. In every triangle, the sum of the three angles is equal to two right angles; in a square, there are four right angles.

5. The sum of all the interior angles of a polygon, is equal to as many times two right angles as there are units in the number of sides less two.

NOTE.—A re-entering angle is one whose vertex is directed

inward, and a salient angle has its vertex directed outward.

6. Every chord is less than the diameter.

7. A straight line cannot meet the circumference of a circle in more than two points.

8. The circumference of a circle may be made to pass through any three points which are not in a right line, but the circumference of only one circle may be made to pass through the same points.

9. If the distance of the centres of two circles is equal to the sum of their radii, these two circles will touch each other externally.

10. If the distance of the centres of two circles is equal to the difference of their radii, these two circles will touch each other internally.

11. In the same circle, or in equal circles, if two angles at the centre are to each other as two entire numbers, the intercepted arcs will be to each other as the same numbers.

12. Whatever may be the ratio of two angles, these two angles will always be to each other as the arcs intercepted between their sides, and described from their vertices, as centres, with equal radii.

13. The inscribed angle has for its measure the half of the arc comprehended between its sides.

14. The angle formed by a tangent and a chord, has for its measure the half of the arc comprehended between its sides.

15. In a right-angled triangle, the square of the longest side is equal to the sum of the squares of the other two sides.

16. Every triangle is half of a parallelogram of the same base and altitude.

17. The area of a trapezoid is equal to the product of its altitude by half the sum of its parallel sides.

18. A line drawn so as to divide a triangle parallel to its base, divides the sides proportionally.

19. If from the right angle of a right-angled triangle the perpendicular be let fall upon the hypothenuse—

I. The two partial triangles will be similar to each other, and to the whole triangle.

II. Either side will be a mean proportional between the hypothenuse and the adjacent segment; and—

III. The perpendicular will be a mean proportional between the two segments.

20. Two triangles, which have an angle in the one equal to an angle in the other, are to each other as the rectangles of the sides, which contain the equal angles.

21. Two similar triangles are to each other as the squares of their homologous sides.

22. Two similar polygons are composed of the same number of triangles, which are similar to each other, and similarly disposed.

23. The perimeters of similar polygons are as their homologous sides, and their surfaces are as the squares of these sides.

24. The product of three sides of a triangle is equal to the surface multiplied by double the diameter of the circumscribed circle.

25. The surface of a triangle is equal to its perimeter, multiplied by half of the radius of the inscribed circle.

26. The two diagonals of an inscribed quadrilateral figure, are to each other as the sums of the rectangles of the sides adjacent to their extremities.

27. The circumferences of circles are as their radii, and surfaces are as the squares of their radii.

28. Two angles at the centre, measured in two different circles, are to each other as the contained arcs divided by their radii.

29. The capacity of solids are as the cubes of their properties.

30. The convex surface of a cone is equal to the circumference of its base, multiplied by half its slant height.

31. The convex surface of the frustum of a cone, is equal to its side, multiplied by half the sum of the circumferences of the two bases.

N.B.—For demonstrations of the preceding problems, the student is referred to the Cambridge Mathematics.

PROBLEMS, RULES AND EXAMPLES.

N.B.—The more common Problems in Mensuration excluded in this, will be found in most works on Arithmetic.

An application of the following Problems and promiscuous questions, with the more common Problems just mentioned, it is believed, contain precedents for solving the great variety of questions generally classed with Mensuration. Also, see "*Sundry Tables*," IV. *and* V.

PROB. I.—*To estimate the Distance of Objects on level ground, or at sea, having only the Height given.*

RULE. 1—To the earth's diameter, (viz., 42056462 feet,) add the height of the eye, and multiply the sum by that height, and the square root is the distance required.

2. The required distance is a tangent to the earth's surface; therefore, to find the distance of two elevated objects, when the right line joining them touches the earth's surface between those objects, work for each object separately, and the sum of the square roots of the products is the distance of the two objects from each other.

EXAMPLE.

How far may a mountain be seen on level ground, or at sea, which is a mile high, supposing the eye of the observer elevated five feet above the surface?

OPERATION.

$\sqrt{(42056462+5,\times 5)}=$ - - - - 2,746 miles.
$\sqrt{(42056462+5280,\times 5280)}=$ - 89,253 "

Ans. 91,999 "

PROB. II.—*To estimate the Height of Objects on level ground, or at sea, having only the Distance given.*

RULE. 1—From the given distance, take the distance which the elevation of your eye above the surface will give, found by the last Problem.

2. Divide the square of the remainder, in feet, by 42056462 feet, and the quotient will be the height required.

EXAMPLE.

Being on my return from a foreign voyage, and finding by my reckoning I was 5¼ leagues from Portland light-house, it

being in the dusk of evening, with my telescope, I descried the lamp of the light-house in the horizon, at which time my eye was elevated 6 feet above the surface of the water. What is the height of the light-house above the water?

OPERATION.

$5\frac{1}{2}$ leagues=16.5 miles.

Then $16.5-\surd(42056462+6, \times 6)=13.943$ miles, equal 73619 feet, nearly.

Then, $73619\times73619, \div 42056462=129$ feet, nearly, Ans.

PROB. III.—*To determine a right-angled triangle, whose base coincides with the diameter of a given semi-circle, about which it is circumscribed; the ratio of the hypothenuse and perpendicular being given.*

RULE.—Take any two numbers in the same ratio of the hypothenuse and perpendicular; then, as the hypoth. is to sine 90, so is the perpend. to sine of the angle opposite. And as radius given to the semi-circle is to sine 90, so is tang. double the angle first found to the actual length of the perpendicular, from which, by means of the given proportion, the other sides may be found.

PROB. IV.—*To Measure a Triangle.*

RULE.—From half the sum of the three sides, subtract each side severally; multiply these three remainders and the said half sum continually together; the square root of the last product will be the area of the triangle.

PROB. V.—*To Measure any irregular plane figure.*

RULE. The whole may be divided into triangles, and measured separately; the sum of the area of the triangles will be the area of the whole.

PROB. VI.—*Given, the minutes of a Survey, to draw a Map of it, and to find the area by Construction.*

RULE.—Draw a line to represent a Meridian, from which lay off a Bearing or Course of the first side of the field, from a line of Chords; and from a scale of equal parts measure the length of the side, and draw a line to represent it; at the end of this line, draw a line parallel to the meridian line, and then lay off the second side of the field, as before taught; proceed in the same manner to draw parallel lines, and to lay off the several sides, till the whole is protracted; divide this map into tri-

angles, by drawing diagonals, and the sum of the areas of these triangles is the area of the field.

Note.—In protracting a survey, let the top of the paper be considered as N., the bottom S., the right hand as E., and the left hand W.; lay the course to the right or left of the meridian line, according as it is E. or W., and from the upper or lower part of the line, according as is N. or S.

N.B.—To find the area of a field, by calculation, the sides and angles of oblique-angled triangles, belongs more properly to what is contained in some work on Surveying, and consequently for this and other reasons will be omitted in this.

Prob. VII.—*Given, the Segment of a Circle, to find the Length of the Arch.*

Rule.—Divide the segment into two equal parts; then measure the chord of the half arch, from the double of which subtract the chord of the whole segment; and one-third of that difference being added to the double of the chord of the half arch, will give the length of the arch line.

EXAMPLE.

The whole chord of a segment is 216, either of the other sides is 126. Required, the arch line.

* 126×2,—216÷3,+252=264, length of the arch line.

Prob. VIII.—*Given, the Chord and versed Sine of a Segment, to find the Diameter of a Circle.*

Rule.—Multiply half the chord by itself, and divide the product by the versed sine; add the quotient to the versed sine; the sum will be the diameter.

N.B.—The diameter of a circle to its circumference, is as 1 to 3.14159265358979323846264338387950288419716939937 51058209749445923078164062862089986280348253421170 679, nearly. Decimals 100 in number.

To find a square that is exactly equal to a given circle, has never yet been discovered, and *probably never will.*

Prob. IX.—*To find the length and breadth of a parallelogram, when the area is given, and the length exceeds the breadth by a certain number of rods.*

Rule.—Square half the number of rods that the length ex-

ceeds the breadth, add the square to the area, and to the square root of the sum add the half number of rods which the length exceeds the breadth; the sum is the length; subtract the difference of the two sides from the length; this difference is the breadth.

PROB. X.—*The diagonal and length of a parallelogram being given in one sum, and the breadth separately, to find the length and area.*

RULE.—Divide the square of the breadth by the sum of the diagonal and length; the quotient is the excess of the diagonal above the length, half of which excess, subtracted from half the sum of the diagonal and length, will leave the length, which multiplied by the breadth, gives the area.

PROB. XI. *The diagonal and the breadth of a parallelogram being given in one sum, and the length separately, to find the breadth.*

RULE.—From the square of the diagonal and breadth, subtract the square of the length, divide the remainder by twice the sum of the diagonal and breadth, the quotient will be the breadth.

PROB. XII.—*Given, the diagonal and area of a parallelogram, to find the length and breadth.*

RULE. 1.—Divide the area by the diagonal, and square the quotient. Square half the diagonal, and the square root of the difference of the squares, subtract from half the diagonal.

2. Square the remainder, and add the square of the quotient, and the square root of the sum of the two squares will be the breadth, by which divide the area, the quotient is the length.

PROB. XIII.—*To measure a Cylinderoid, or Prismoid.*

RULE.—To the areas of both bases, add a mean area, that is, the square root of the product of the two bases, multiply that sum by a third of the height or length, the produet is the solidity.

PROB. XIV.—*To find how large a Cube may be cut from any given sphere, or be inscribed in it.*

RULE.—Extract the square root of one-third the square of the diameter, the root is the side of the required cube.

PROMISCUOUS QUESTIONS IN MENSURATION.

1. What is the Hypothenuse and Perpendicular of a right-angled triangle, whose sides are in the ratio of 7 to 4, the base being 100?

* $\sqrt{7\times7,-4\times4}=5.7445$, ratio of the base.

$$\text{Then,}\left\{\begin{array}{l}5.7445:4::100:\ 69.68,\text{ Perpendicular,}\\ \quad\text{“}\quad :7::\ \text{“}\ :121.94,\text{ Hypothenuse,}\end{array}\right\}\text{Ans.}$$

2. In an olique-angled triangle, the product of the two sides is 186, and their difference 3.5, the shortest side is to the base in the ratio of 4 to 7. Required, the sides.

* $3.5\times3.5,\div4=3.0625,+186=189.0625$; its square root is 13.75.

Then, $13.75-3.5\div2=12$, shortest side.
And, $13.75+1.75=15.5$, longer side.
And, if $4:12::7:21$, base.

3. In a right-angled triangle, the difference of the sides is 70 rods, and the difference of the segments of the hypothenuse, made by a perpendicular let fall from the angle opposite, is 98. Required the sides.

* In questions of this nature, the difference of sides will form the hypothenuse, and the difference of the segments the sum of the two legs of a triangle, in all respects similar to the given one.

Therefore, $98\times98,-70\times70=4704$.
$\sqrt{70\times70-4704}=14$, difference of sides.
And, $98+14,\div2=56$; also, $98-14,\div2=42$.

$$\text{Then, as}\left\{\begin{array}{l}14:70::56:280,\text{ Base,}\\ 14:70::42:210,\text{ Perpendicular,}\\ 14:70::70:350,\text{ Hypothenuse,}\end{array}\right\}\text{Ans.}$$

4. In a triangle, the area is 216, the angle at the base 36° 52′, and the cube of the sides is 46656. Required, the sides.

* The base, multiplied by the perpendicular, must be 432.

As the sine of the given angle is to 3, so is the co-sine of the same angle to 4, calling the base 4, and the perpendicular 3; and $4\times4=16$; also, $3\times3=9$; and $4\times3=12$, twice the area.

$$\text{Then, as}\left\{\begin{array}{l}12:16::432:576,\sqrt{576}=24,\text{ Base of Area,}\\ 12:\ 9::432:324,\sqrt{324}=18,\text{ Perpendicular}\\ \sqrt{(24\times24+18\times18)}=30,\text{ Hypothenuse,}\end{array}\right\}\text{Ans.}$$

5. There are two columns in the ruins of Persepolis, left standing upright; one is 70 feet above the plane, and the other 50 ft.; in a straight line between these, stands a statue, 5 feet in height; the head of which is 100 feet from the summit of the higher, and 80 feet from the top of the lower column. Required, the distance between the tops of the two columns.

* $100\times100,-\overline{70-5}\times\overline{70-5}=5775$; $\sqrt{5775}=75.99342$.
$80\times80,-\overline{50-5}\times\overline{50-5}=4375$; $\sqrt{4375}=66.143782$.
$\overline{75.99342+66.143782}|^2=20202.984192388804$.
$70-50=20$; $20\times20=400$.
$\sqrt{(20202.984192388804+400)}=143.5373396$, feet, Ans.

6. Admit 10 hhds. of water are discharged through a leaden pipe, of $2\frac{1}{2}$ inches diameter, in a certain time. What must be the diameter of another pipe that shall discharge four times as much in the same time?

* $2\frac{1}{2}\times2\frac{1}{2}=6\frac{1}{4}$; $6\frac{1}{4}=6.25$, and 4 is the given proportion.
$6.25\times4=25$; then, $\sqrt{25}=5$ inches, Ans.

7. Two wheels of unequal dimensions, made fast to an axle 11 feet long, are set to rolling on an even plain; the path described by the less wheel, encloses just 314.16 square rods; and the less wheel is 5 feet in diameter. Required, the diameter of the greater.

* $\sqrt{(314.16\div.7854)}=20,\times16\frac{1}{2}=330$ feet, diameter of the 314.16 square rods; $11\times2=22$.

Then, $330+22=352$ feet, diameter of the circle formed by the rolling of the large wheel.

Then, as 330 feet : 5 feet : : 352 feet : $5\frac{1}{3}$ feet, diameter of the greater wheel, Ans.

8. In turning a chaise within a ring of a certain diameter, I observed the outer wheel to make two turns, and the inner wheel to make but one; both wheels were four feet high. If both were fixed to an axle 5 feet asunder, what was the circumference described by the outer wheel?

* By the question, the outer ring must be twice the diameter of the inner ring. The distance between the rings being 5 feet, it follows that the diameter of the inner ring will be 10 feet, and the diameter of the outer ring 20 feet.

Then, if 113 : 355 : : 20 : 62.83, &c. feet, Ans.

9. An orchard of 2400 mulberry trees were arranged that the

length was to the breadth as 3 to 2; and the distance of each tree one from the other, 7 yards. Required, the number in length and breadth, and the square yards of ground occupied.

* Rule.—As 2:3::2400:3600; $\sqrt{3600}=60$ trees in length.

As 3:2::2400:1600; $\sqrt{1600}=40$ trees in breadth.
And, $60-1,\times 7=413$; also, $40-1,\times 7=273$.
Then, $413\times 273=112749$, square yards occupied.

10. Three men are to carry a stick of timber 12 feet long, and of equal size from end to end; one man is to carry the hind end, and two are to carry the forward end with a lever. How far from the forward end must the lever be placed, that each man may sustain an equal portion of the weight?

* Two men sustain twice as much, if near the forward end, as the man at the hind end; therefore, the two should be twice as near the centre; consequently, 3 feet, Ans. Hence, if a stick 16 feet long was to be carried by five men, four men forward and one behind, the forward men must be one-fourth as far from the centre, viz., 6 feet.

11. Suppose a pole to stand on a horizontal plane, 75 feet in height above the surface, at what height from the ground must it be cut off, so that its top may fall on a point 55 feet from the bottom of the pole, the end, where it was cut off, resting on the upright part?

* Rule.—$75\times 75,-55\times 55,\div 75\times 2=17\frac{1}{3}$ feet, Ans.

12. A pole, 80 feet long, and perpendicular to a level plane, was broken off; the top of the broken part touching the plane, the other end resting on the stump, the parts making an angle of 37 degrees with the horizon. What is the height of the stump?

* The natural sine of $90°-37°=.7986355$; and the natural sine of $90°=1$.

Then, as 1.7986355 : 80 : : .7986325 : 35.521238, &c. feet, Ans.

N.B.—The numbers just used as the representatives of the number of degrees mentioned in the question, may be found in Mathematical Tables, constructed for the purpose of making calculations by the representatives of the given numbers. Those tables belong to another work.

13. A traveled N. 8 miles an hour, and B, E. 6 miles an hour. Required their distance asunder in each hour.

*$\sqrt{8\times8,+6\times6}=10$ miles in one hour; $10\times2=20$ miles in two hours, &c. Ans.

14. Suppose a ship to sail from lat. 43° N. between N., and E., till her departure from the meridian be 45 leagues, and the sum of her distance and difference of lat. to be 135 leagues. I demand her distance sailed and latitude come to ?

*$135\times135,-45\times45,\div135\times2=60$ leagues, and 60×3 equal 180 miles, which is equal to 3 degrees, difference of latitude; then, $135-60=75$ leagues, distance sailed. And, as the vessel is sailing from the equator, the latitude is increasing; therefore, $43°+3°=46$ degrees N. is the latitude.

15. Two ships, A and B, sail from the same port; A sails the first day between S. and E., 105 miles; B sails S. 147 miles the first day, and wishing again to meet A, she shapes her course between S. and E.; A continues the same course and both vessels sail alike. At the end of the second day they meet. Required the distance and course run by each vessel.

*This is similar to the third question, 105 being the difference of the sides, and 147 the difference of the segments.

Then, as 105 : 147 : : the distance sailed by A, to the latitude and departure.

To form the like figure, $105\times105\times2,-147\times147=441$.

$\sqrt{441}=21$; and $147+21,\div2=84$, longest side.

Again, $147-21,\div2=63$, less aside.

Then, as 21 : 84 : : 84 : 420 miles, distance sailed by A.

And, as 21 : 84 : : 63 : 315,+147, distance sailed by B.

Consequently, A sailed S. 36° 52′ E., 420 miles.

And B, S. 147 miles, then S. 53° 8′ E., 315 miles.

16. From the top of a steeple 165 feet high, the angle of depression of the nearest bank of a river is 11° 15′, that of the opposite bank is 6° 15′. Required the width of the river.

*Let S. signify *sine*, L. *logarithm*.

Then, as S. 11° 15′ : L.165 : : radius, to the distance from the top of the steeple to the nearest bank of the river.

And, as S. 6° 15′ : L. of the distance last found, so is S. 5° (11°15′—6° 15′), to the L. of the width of the river, 41. 13 rods, Ans.

N. B. The L. of a given quantity is its representative—See "*Flint's Survey*," pages from 149 throughout; also, the article "*Trigonometry*" in the same work.

17. At a certain point I took the elevation of a tower, 3°15′, then measured towards the tower on the angle of depression, 7°, 333 feet to a level with the base of the tower; when I took the elevation again, 8°. Required the height of the tower, and the distance from the second place of observation to the base; also, how much higher was the land at the first place of observation, than at the second.

*180°—8°+7°=165°, angle opposite the side or line from the first station to the top of the tower.

And, 165°+7°+3° 15′=175° 15′; 180°—175° 15′=4° 45′, the third angle of the first triangle.

Then, as S. 4°45′ : L. 333 : : S. 10° 15′ : 715.5 feet, L. of the distance from the second station to the top of the tower.

And, as radius : L. 715.5 : : S. 8° : height of the tower, (99.6 feet.)

Also, as radius : L. 715.5 : : co—S. 8° : distance from the second place of observation to the base, (708.6 feet.)

Then as S. 10° 15′ : L. 715.5 : : S. 165° : 1041 feet nearly, the distance from the first station to the top of the tower.

Lastly, as radius : L. 1041 : : S. 3°15′ : L. of a number of feet to be subtracted from the height of the tower, the remainder will be the difference in the height of land, (40.58 feet.)

18. Two persons made observations on the altitude of a meteor, both being on the same side of it, and in a vertical plane passing through it. The distance between their stations was 200 rods, and at one, the angle of elevation was 36° 25′, at the other, 32° 50′, and at the last station, the disk of the meteor subtended an angle of 5′. Required the distance from the last station, also its height and diameter.

*I. 180°—36° 25′=143° 35′, angle formed by the stationary distance and line from the second station to the meteor.

II. 143°35′+32°50′=176° 25′; 180°—176°25′=3° 35′, third angle of the first triangle.

III. As S. 3°32′ : L. 200 : : S. 32°50′ : L. of the distance required, (5 M. 3 Q. 60 R.)

IV. 180°—2′=179°58′, ÷2=89°59′. Then, by *Natural Sines*, (for greater accuracy.)

As S. 89° 59′ : 1990 rods (the distance) : : S. 2′, to the diameter of the meteor, (45ft, 5½in.)

V. Its height is 3 M. 70 R. and by a little attention to the preceding parts of the solution, the method for obtaining it will be very obvious.

19. An Eliptical Garden contains 160 square rods. Required the dimensions of another garden of the same or like proportion, containing half an acre; the transverse diameter of the smaller garden to be the same as the conjugate diameter of the larger garden.

*The ratio of the conjugate to the transverse is as 1 to 1.4142.

Then, $\sqrt{80} \div 1.4142 \times .7854 = 8.4868$, conjugate diameter.

And, $8.4868 \times 1.4142 = 12.00203256$, transverse diameter.

20. A circular garden containing one acre, one quarter, and one rod, has a gravelled walk on the outer side of it within the circle, that takes up twelve rods of ground. Required its diameter and the width of the walk.

*$\sqrt{}$*Its area*$\div .7854$=about 16 rods, diameter of the garden.

And $201-12=189$; $\sqrt{189 \times ,7854}$=diameter of the enclosed circle, which subtracted from the diameter of the garden, and half the sum taken, leaves the width of the walk, 4 feet, Ans.

21. A frustum of a pyramid 50 feet high, is 12 inches square at the base, and 2 inches square at its top, and I wish to take off from the top end, seven-two-hundred and fifteenths of its whole area; at what height from the bottom must it be cut?

*I. It may be seen that either of its four sides is an isoceles triangle, and a given portion of its whole area is to be taken off by a line parallel to its base; therefore, first, complete the height of the frustum, thus:

As 12in.—2in. : 50ft. : : 12in. : 60 feet, height of the *pyramid.*

II. Obtain the diagonal of the surface of the base, half of which square and add to the square of the perpendicular, extract the square root of this sum, the root is a side of either of the four triangles.

III. See Principles 21 and 31, (PRINCIPLES ASSUMED, MENSURATION); therefore, as the area of one of the triangles is to the square of its sides, so is the area of the part to be taken off to the square of the sides that contains it. By a due consideration of the preceding, it is presumed that further explanation is entirely unnecessary.

N. B.—The last example involves a principle which has often perplexed both teacher and scholar, failing to obtain correct results by operating on wrong principles.

22. A square pyramid, each side of whose base is 30 inches, and perpendicular height 120 inches, is to be divided by sections parallel to its base into three equal parts. Required the perpendicular height of each part.

*Its content is 360000 solid inches, $\frac{2}{3}$ of which is 2400 and $\frac{1}{3}$ equal 12000 solid inches.

Then, as $36000 : \overline{120}|^3 : : \left\{ \begin{matrix} 24000 : 1152000 \\ 12000 : \ 576000 \end{matrix} \right\}$ the cube root of each respectively, being 104.8, and 83.2.

Then, 120 less 104.8=15.2, length of the thickest part. And 104.8—83.2=21.6, length of the middle part; consequently, 83.2 inches is the length of the top part.

23. A cooper having a cask 40 inches long and 32 inches at the bung diameter, is ordered to make another cask of the same shape, but which shall hold just twice as much. What will be the bung diameter and length of the new cask?

$\left.\begin{matrix} *\sqrt{}(40\times40\times40\times2)=53.3 \text{ \&c. inches, its length,} \\ \sqrt{}(32\times32\times22\times2)=40.3 \text{ \&c. inches, its diameter,} \end{matrix}\right\}$ Ans.

24. There is a box 4 feet wide, 4 feet high, and 4 feet long. What is the side of another box of the same shape that will contain $\frac{1}{8}$ of the quantity?

*$4\times4\times4,\div8=8$, and $^3\sqrt{}8=2$, Ans. 2 feet. Hence we learn that the capacity, properties, &c. of solids are as the cubes of those properties.

25. A, B and C join to buy a grindstone of 36 inches diameter, which cost $\$3\frac{1}{3}$, and towards which A paid $\$1\frac{1}{6}$, B $\frac{8}{20}$ of its cost, and C the remainder. The waste hole for the spindle was five inches square. To what diameter ought the stone to be worn, when B and C severally begin to grind with it allowing for the spindle, and A first grinding down his share, next B, and then C?

*$\$1\frac{1}{6}=\frac{7}{20}$; $\frac{8}{20}=\$1\frac{1}{3}$; C's share to pay is $83\frac{1}{3}$ cents.

Thus, $\frac{8}{20}+\frac{7}{20}=\frac{15}{20}=\frac{3}{4}$, $\frac{4}{4}-\frac{3}{4}=\frac{1}{4}=\frac{5}{20}$, C's $\sqrt{}5\times5\times2=7\frac{1}{14}$, diameter of a circle that will circumscribe a 5 inch square. $7\frac{1}{14}\times7\frac{1}{14}\times11,\div14=39.285$, area of that circle whose diameter is $7\frac{1}{14}$ inches.

Area of its side is its diameter, $36\times36\times.7854=1017.8784$.

$\sqrt{}(1017.8784-1017.87847\times7\div20,+39.285\times7\div20)$ is 29.324 inches in diameter when B begins to work, Ans. 1.

And, $(\sqrt{}1017.8784-1017.8784\times3\div4,+39.285\times3\div4)$ is equal to 19.013 &c. inches, diameter when C. begins to work, Ans. 2.

26. For a job a tinker appeared in a nettle,
So I asked him to make a flat-bottomed kettle;
Let the top and the bottom diameters be,
In just such proportion as five is to three;
Twelve inches in depth I proposed and no more;
And to hold in ale gallons seven less than a score.

Required the diameters.

*Rule. 1.—Reduce the given numbers of gallons to cubic inches.

2. Multiply the numbers together, which express the proportion of the diameters, and to three times their product add the square of the difference of those numbers; then multiply this sum by .7854, and the product by $\frac{1}{3}$ of the depth.

3. Divide the number of cubic inches which the vessel is to hold by the last product, and extract the square root of the quotient.

4. Multiply this square root by each of the numbers expressing the proportion of the diameters, separately, and the products will be the diameters required, viz, 24.40, &c. inches, larger diameter; and 14.64, &c. inches, smaller diameter.

27. The diameter is required of a circular aperture at the bottom of a cistern, whose depth is 10 feet and length and breadth each 20 feet, that will empty it when full of water in 15 minutes?

*$\sqrt{10}=3.162, \times 2.065=6.52$, cubic feet discharged through a square inch hole in one minute, if kept full, which would be double the quantity of water that it will, if it empties itself in the same time. (See *Enfield's Philosophy* and *Ree's Cyc. Art. Water.*)

Therefore, its content, $4000 \times 2, \div 6.52=1230$ minutes, time in which 8000 feet will be discharged through an aperture an inch square, if the cistern be kept full, and it empties itself in the same time, through the same aperture. And, inversely, as 1230min. : 1 inch : : 15min. : 82 inches, area of the required diameter; hence, $\sqrt{82 \times .7854}=10.12$in., Ans.

28. Given, the area of a triangular field equal to 8 acres and 2 rods. Required, the triangle when the area of the greatest inscribed circle is equal to half the field.

* This question belongs to the *indeterminate analysis*. As it is not limited to any particular triangle, I shall give the solution which belongs to the right angle.

The area in rods, $1282 \div 2, \div .7854$, and the square root of

the last quotient, equal to 28.568,$\div$2=14.284, radius of the inscribed circle.

Then, 1282—14.284$\times$14.284 = 1077.967344.

1077.967344,$\div$ 14.284=75.466.

$\surd$(75.466$\times$75.466,—1282$\times$4)=23.814.

75.466+23.814,$\div$2=49.64,+ 14.284 = 63.924, the base.

75.466—23.814, $\div$2=51.652,+ 14.284= 40.11, required perpendicular. From these obtain the Hypothenuse at pleasure.

29. Required each side separately, and the area in acres of a triangular field; the sum of the three sides of which is 250 rods, and their product 549486.

*This is a question of *indeterminate analysis*. I find by trial that one of the sides of the triangle is between 60 and 65, but there is no number between these two which is a measure of 549486, except 63; I therefore assume 63, as one of the sides.

Then, 549486$\div$63=8722, and 250—63=187.

Then, $\surd$(187$\times$187,—8722$\times$4)=9, and 187+9,$\div$2=98 another of the sides. And, 187—9,$\div$2=89, the other side. Hence 17 acres 24 rods, &c., Area.

30. There is a section of a tree 25 inches over; I demand the difference of the inscribed and circumscribed squares, and how far they differ from its area.

*25$\times$25,—12.5$\times$12.5$\times$2=312.5 inches, difference of the squares. 25$\times$25,—25 $\times$25$\times$11,$\div$14=134.125 inches, the circumscribed square, in area more than the section.

25$\times$25$\times$11,$\div$14,—12.5$\times$12.5$\times$2 = 178.375 inches, the inscribed square, less than the area of the section.

31. A certain lady, the mother of three daughters, has a farm of 785$\frac{5}{7}$ acres, in a circular form, with her dwelling house in the centre. Being desirous of having her daughters settled near her, she gave to them three equal parcels, as large as could be enclosed in three equal circles, included within the periphery of her farm, one to each, with a dwelling house in the centre of each. How many acres did the farm of each daughter contain; how many acres did the mother retain; how far apart were the dwelling houses of the daughters; and how far was the dwelling-house of each daughter from that of the mother?

*I. The diameter of the lady's farm is 400 rods or 100 four rod chains. And $\surd$(100$\div$2$\times$100$\div$2,—25$\times$25)=43.30127.

II. Again, 43.30127,+50=93.30127.

III. As 93.30127 : 50 : : 43.30127 : 23.20508, $\times 2 = 46.41016$ chains, diameter of either of the daughters' farms, and the distance of their houses from each other.

EXPLANATION.—Half the diameter of the periphery is the radius of the chord of 60 degrees; half the radius is equal to the versed sine of half the arch of 120 degrees; and the square root of the difference between the squares of the radius and the versed sine, is equal to the sine of half the arch or half the chord of 120 degrees. As the sum of the radius and half the chord of 120 degrees, is to the radius, so is the said half chord to the semi-diameter of a circle inscribed within two radii and an arch of 120 degrees of the given circle.

IV. Chains $100 \div 2, - 46.41016 \div 2 = 26.79492$ chains, distance of the lady's dwelling house from those of her daughters.

V. It is presumed that further explanation is wholly unnecessary.

32. A and B bought a cheese of equal thickness; its diameter 20 inches; they gave $3 for it; A paid $1.30; B paid the remainder. It is required to divide it between them by a chord line, in proportion to what each one paid.

*As 3 : .7854 : : 1.30 : .34034, which by the table of the areas of circular segments, adapted to a circle whose diameter is unity, we find .447545 corresponds to the versed sine of the required chord.

Hence, as 1 : .447545 : : 20 : 8 9509 inches, versed sine of the chord.

Then, radius, 10 less versed sine, $8.9509 =$ cosine, 1.0491. Lastly, $\sqrt{(10 \times 10, - 1.0491 \times 1.0491)} = 9.9448, \times 2 = 19.8896$ inches, length of the required chord.

33. A tract of land containing 100 acres, was purchased by A and B for $500, in the payment of which they paid equal sums of money. In the division of it, A, wishing to have his share off the side adjoining his own farm, agreed to receive so many acres less than one half as would make his part 75 cents more per acre than B's. How many acres had each, and what was the price per acre?

*RULE 1.—Multiply the whole number of articles, acres, *or anything else*, by the difference in the two prices of one article, and subtract the product from the whole sum or price of the whole. Then divide the remainder by twice the difference in the price of one article, calling this the *first quotient*.

2. Multiply half the whole sum by the whole number of articles, and divide the product by the difference in the prices of one article, calling this the *second quotient.*

3. To the square of the first quotient add the second quotient, and extract the square root of the sum ; and from the square root subtract the first quotient, and you will have the share of one, or the larger share.

Subtract this share from the whole number of articles, and you will have the share of the other, or the smaller share.

OPERATION.

$500-100\times.75,\div.75\times2=283.333$, *first quotient.*

And $500\div2\times100,\div.75=33333.3333$, *second quotient.*

$\sqrt{(283.\overline{33}|^2+33333.3333)}=337.06,-283.33=53.73$ acres, B's share.

And $100-53.73=46.27$ acres, A's share; consequently, \$5.40, A's per acre, and \$4.65 B's.

34. A merchant tailor bought 40 yards of cloth $2\frac{1}{4}$ yards wide, but being wet it shrunk in length upon every 5 yards $\frac{1}{8}$ of a yard, and in width 1 nail on every yard. To line this cloth he bought baize 5 quarters wide, which being made wet, did shrink the whole width on every 20 yards in length, and in width it shrunk half a nail. Required the number of yards of baize, used in lining this cloth.

*If 4yds. : 2n. : : 40yds. : 1yd. 1qr.

And, yds. 40—1yd. 1qr.=38yds. 3qrs., length of the cloth after shrinking; also, 2yds. $1\frac{3}{4}$n., its width after shrinking.

Then, $38.75\times2.0109375=77.923828125$ square yards remains.

Again, yds. of baize 20×5qrs.=25yds. before it shrinks, and $18.75\times1.21875=22.8515625$yds. after it shrinks. Then, if .25 : 22.8515625 : : 77.923828125 : 71.227 &c. yards, Ans.

35. A, B and C purchase a farm of 95 acres, for \$1250, of which C takes 12 acres at cost. A advances \$400 and B \$500, agreeing to share the 83 remaining acres in proportion to their advance. Required the cost of C's part, the quantity of land that A, or B has, and its cost.

*1250—500+400=\$350, C advanced.

\$1250÷95=\$$13\frac{3}{19}$ per acre.

\$$13\frac{3}{19}\times12=$\$$157\frac{17}{19}$; 350—\$$157\frac{17}{19}$=\$$192\frac{2}{19}$, C has left.

500+400+$192\frac{2}{19}$=\$$1092\frac{2}{19}$ A, B and C have left, which A and B assume in proportion to their advance.

As $900 : 1092\frac{2}{19} : : 400 : \$485\frac{65}{171}$, cost of A's land; hence, $36\frac{38}{45}$ acres; consequently, $\$606\frac{124}{171}$, cost of B's land and $46\frac{7}{45}$ acres.

36. The base of the triangle is 54 and the sum of the two sides 66. Required the two sides when the angles at the base are as 1 to 3.

**Rule by symbols.*—Let $a=60$, (half the sum of all the sides of the triangle,) $n=33$, (half the sum of the two sides,) $m=a$ less n, $=27$ $d.=54$, and $x=$half the difference of the two unknown sides.

Then, by the principles of Plain Trigonometry, I obtain the following equation:

Thus, $\left\{ 4 \left\{ \frac{a\,(m+x)}{d\,(n+x)} \right\} -3 \right\}^2 = \left\{ \frac{m-x}{n+x} \right\} \times \left\{ \frac{n+x}{n-x} \right\}$; the resolution of which gives $x=14.3519$; hence, the sides are 47.3519 and 18.6481. See principles 12, (PRINCIPLES ASSUMED, MENSURATION.)

36. Required the solid contents of the middle zone, or frustum of a sphere, the axis of which is 12 inches; the shortest perimeter of the frustum being 36.34567419 inches.

*As $355 : 36.34567419 : : 113 \div 2 : 5.784593$.

And, $5.784593, \times 5.784593 = 33.461518$ &c.

Then, $\sqrt{(6 \times 6 - 33.461518 \text{ \&c.})} = 1.59326, \times 2 = 3.1865$ inches.

And, $12 \times 12 \times 2, +11.569186 \times 5.784593 \times 2 = 421.840074$

Lastly, $421.840074, \times .7854, \times 3.18652, \div 3 = 351.917266$ &c. ,cubic inches, Ans.

37. There is a conical glass, 6 inches high, 5 inches wide at the top, and 6 inches deep; it is $\frac{1}{6}$ part filled with water. What must be the diameter of a ball, let fall into the water, that shall be immersed by it?

*$\frac{1}{2}$ of $5=2.5$; $\sqrt{(2.5 \times 2.5 + 6 \times 6)} = 6.5$, length of one side of the glass.

Then, call its width at the top, A B, half its width, A D, its depth, D C, the length of one side, A C, the periphery of the ball touching a point F, on the side A C, D F H G, and its centre, E; and draw the figure at pleasure, without any regard to precision, observing to let each of the sides and its width touch the periphery, D F H G. Because A D E F is a regular figure; and the angles A D E and A F E being equal, each being a right angle, and the sides D E and F E, being also equal, because they are radii of the circle D F H G, the sides A D

and A F are also equal. Hence, A C being 6.5 inches, and A F 2.5 inches, F C will be 4 inches; 6.5—2.5=4 inches. Then, by similarity of triangles, proceed thus:

As 6in. : 2.5in. : : 4in. : $1\frac{2}{3}$, F E. $1\frac{2}{3}\times2=3\frac{1}{3}$, F G$=\frac{10}{3}$in.

Then, $5\times5\times.785398\times2=39.2699$ inches, contents of the cone. And, $\frac{10}{3}\times\frac{10}{3}\times\frac{10}{3}\times.5236=19.3925$ inches, contents of the sphere. The content of cone, less that of the sphere, equal 19.8774 cubic inches of water that will remain in the cone after the sphere is immersed. Since regular solid bodies are to each other as cubes of their homologous sides, I say,

As the quantity of water it requires to immerse the sphere in the *given* cone is to the cube of the diameter of the sphere, so is any other quantity of water in the conical glass to the cube of the diameter of a sphere that may be immersed in it. Now, the quantity of water given to immerse the *required* sphere is $\frac{1}{5}$ of the content of the conical glass, being $\frac{39.2699}{5}=7.85398$ &c. cubic inches. The cube of the diameter of the *given* sphere being $\frac{10}{3}\times\frac{10}{3}\times\frac{10}{3}=\frac{1000}{27}=.027$in.

Therefore, as 19.8774in. : .027 : : 7.85398in. : 14.634114529 &c. inches, the cube root of which is 2.445 &c. inches, diameter of the *required* sphere, Ans.

38. A tract of land is to be laid out in the form of an equal square, and to be enclosed with a post and rail fence, 5 rails high, so that each rod of fence shall contain 10 rails. How large must said square be to contain just as many acres as there are rails that enclose it?

*One mile is 320 rods; $320\times320,\div160=640$ acres; then, $320\times4\times10=12800$ rails.

Then, if 640 : 12800 : : 12800 : 256000 rails; hence, 20 miles square, Ans.

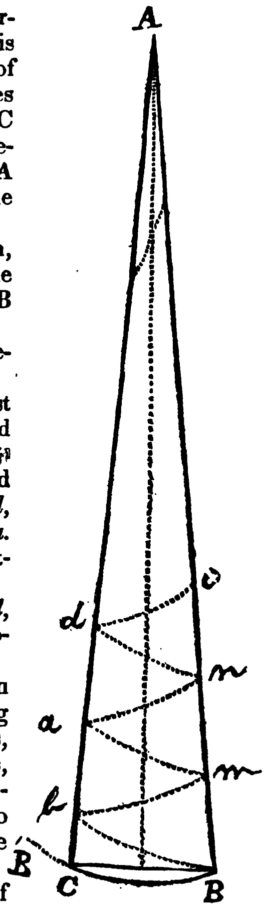
39. Required the area of a circular field surrounded by post and rail fence ten rails high, each rail being a rod in length, and the number of rails equals the number of acres enclosed.

*It may reasonably be supposed, in this and similar cases, that the field consists of as many isoscles triangles as it takes rails to fence it one rail high, because each rail in length helps form the perimeter of a field of more than 60 miles in circumference, therefore the difference of the arc and its chord of a circle of 20 miles or more in diameter, the chord being but a single rod, should be considered as of no value. Hence in this case the number of triangles is a tenth of the required number of rails, or acres. Consequently, each triangle equals ten acres, fenced by ten rails. Therefore, $160\times10\times2$

equal 3200 rods, perpendicular of either triangle; hence, twice the hypothenuse is 6400.000048, &c; accordingly, the area of the field is 201062.4 acres.

40. Required, the length of a thread, winding spirally, once round the height of every three feet of a cone, which is 50 feet in height and 3 feet in diameter at the base.

*The surface of a cone may be exactly represented on a plane. Thus, with the slant height of the cone as radius; say, As the circumference of what A B is radius, is to 360°, so is the circumference of the cone, to the number of degrees in the angle C A B. The angle C A B being taken equal to the degrees thus obtained, the surface C A B will be the surface of the cone spread out on a plane.



The angle C A B being known, A C B will also be known, and the arc C B being known its chord C B is also known.

C *b*, or B *m* will be the distance between the spires or coils.

Hence, B *b*, the spire nearest the base, may be easily calculated by the usual rules of trigonometry.

The spire B *b* may be considered as parallel to *m a*, and *m a* to *n d*, &c. Hence, *b* A : C A : : B *b* : *m a*. But *b* A, *a* A, *d* A, &c. are evidently in arithmetical progression.

Hence, the spires B *b*, *m a*, *n d*, &c., will also be in arithmetical progression.

The first spire B *b*, having been calculated and the last spire being the distance between the spires, which will answer *practical* purposes, we shall have the extremes and number of terms of the progression, to find the sum, which will be the length of the thread required.

N. B.—For precision, instead of

taking the difference between two spires for the last term of the series to be summed, the one previous may be calculated, the sum taken to that point, and then the distance between two successive spires added to the result for the total sum. This would be best in all cases, though more laborious, since this method would give the true result.

PROBLEMS, RULES AND EXAMPLES.

PROB. I.—*The Sum of any two numbers, and their Product given, to find the Numbers.*

RULE.

1. From the square of half the sum, subtract the product, extract the square root of the remainder, the root is half their difference.

2. Add this half difference to half the sum; this sum will be the greater number, which subtract from the whole sum, gives the less number.

EXAMPLE.

Required, the dimensions of a parallelogram, containing one acre and a half, bounded by 64 rods of fence.

* $64 \div 2 = 32$, the sum of one side and one end. $32 \div 2 = 16$. $\sqrt{(16 \times 16, -160 + 80)} = 4$; then, $16 + 4 = 20$, greater number; $16 - 4 = 12$ less number.

PROB. II.—*Given, the Difference of two Numbers, and their Product, to find each number.*

RULE.

1. Square the difference of the two numbers, add one fourth of the square to the product, and twice the square root of this sum is the sum of the two numbers.

2. Then proceed by the latter clause of the rule to PROB. I.

EXAMPLE.

A man had a field in the form of a parallelogram, its

length being 40 rods more than the breadth, and the area 3825 rods. Required, the length and breadth.

* 40 is the difference, and 3825 the product. Therefore, 85 rods, length; and 45 rods, breadth.

PROB. III.—*Given, the Sum of two Numbers, and the Sum of their Cubes, to find each Number.*

RULE.

1. From the cube of the sum, subtract the sum of the cubes, and divide the remainder by three times the sum of the numbers; the quotient will be the product of the two numbers.

2. Then proceed by PROB. I.

PROB. IV.—*Given, the Difference of two Numbers, and the Sum of their Squares, to find each number.*

RULE.

From twice the sum of their squares, subtract the square of their difference. The square root of this sum, will be the sum of the two numbers. Then proceed by PROB. I.

EXAMPLE.

The difference of two numbers is 6, the sum of their squares 3060. Required, each number.

* $\sqrt{(3060\times2,—6\times6)}=78$; then, $78\div2,+6\div2=42$, one number; $42—6=36$, the other.

PROB. V.—*Given, the Sum of two Numbers, and the Sum of their Squares, to find each Number.*

RULE.

From the square of the sum, subtract the sum of the squares, subtract the remainder from the sum of the squares, and extract the square root of this remainder, which will give their difference. Then, proceed by PROB. I.

PROB. VI.—*Given, the Sum and Difference of the Squares of two Numbers, to find each Number.*

RULE.

From the sum take the difference, and half the remainder is the square of the less, which, taken from the sum of the squares, will give the square of the greater; the square root of each, is each number.

PROB. VII.—*Given, the Sum of the Squares of two Numbers, and the Square of their half Sum, to find each Number.*

RULE.

From the sum of the squares, take twice the square of the half sum, and double the square root of half the remainder will be their difference; the square root of the square of their half sum doubled, is the sum of the numbers. Then proceed as before directed.

EXAMPLE.

The sum of the squares of two numbers is 3161, and the square of their half sum is 1560.25. Required, each number.

* $\sqrt{(3161,-156.25\times 2,\div 2)}=4.5,\times 2=9$, difference of the two numbers.

Then, $\sqrt{(156.25=39.5,\times 2)}=79$, sum of the two numbers.

$79-9,\div 2=35$, less number. $79-35=44$, greater number.

REMARK ON SURVEYING.—The author is not aware, that any person but himself has ever discovered the method of obtaining corrected bearings, when local attraction, only, prevents, from the taken bearings, which will, upon their application in calculating the area, work so correctly, that the sum of the northings will equal that of the southings; and the sum of the eastings that of the westings, precisely.

In fact, all authors on surveying treat it as a subject next to impossibility, and the difficulty, for want of a *practical* rule, as the surveyor's greatest perplexity. The following article obviates the difficulty relating to local attraction, and is precisely the rule in question; consequently, it will be of inestimable value to the practical surveyor.

The author has the satisfaction of believing, that he labored not in vain in bringing this intricate matter to a focus, having submitted it to the Literati, and pronounced by them as fully

worthy the notice of those aiming at accuracy in surveying.

Many apologies might be offered for inserting it in *this* work, but the only reason which will be advanced is, *it may do more good than hurt.*

PROB. VIII.—*To expunge local attraction from the bearings of a survey.*

DIRECTIONS.

1. Take the bearings, their reverse, and the distances; and in noting them, let Att. signify *attraction;* B. signify *bearing;* R. B., *reverse bearing;* D., *distance;* Diff., *difference between the B. and R. B.;* and E. or W. at the right of the Diff., signify that the Att. is *east* or *west.*

2. The Diff. of the B. and its reverse shows how much Att. there is.

3. If the B. is greater than the R. B., the Att. should be considered of the same name as the bearing's second letter, but it is of an opposite name if the B. is less than the R. B.

4. In noting the bearings, their reverse, and the distances, note the bearings first, and if S. &c. change them to an opposite name; note their reverse at the right of the bearings, and if S. &c. change them to an opposite name; at the right of the reverse bearings note the Diff.; and note the distances at the right of the Diff.

5. Apply that Diff. which immediately follows the B., having the least or no Att. to the next following B., and as the rule for applying the Att. or Diff. dictates; and if the next Diff. is of the same name as the first applied Diff. their sum should be applied to the next B. to be corrected, but if it is of an opposite name, their difference should be applied, and the same is to be observed in the application of the Att. of any two bearings, calling their difference of the same name as the greater Diff., minding not to apply the last noted Diff. to the first noted B.

RULE.

1. If the bearing to be corrected is a N.W. or a S.E. one, and the Att of the former B. is a W. Att., it should be added to the B to be corrected, but subtracted, if it is a N.E. or a S.W. B.

2. If the Att. is an E. Att., it should be added to the bearing to be corrected, if it is a N. E. or a S.W. one; subtracted, if a N.W. or a S.E. B.

NOTE 1.—The preceding shows the E. Att. to be equal to the W. Att., which will ever be the case, unless an error in the field-work has been committed, which may be detected by observing this fact, and obviated before leaving the field, hence another important advantage.

NOTE 2.—The following are the preceding bearings corrected.

1. B. S. 54° 30′ E.
2. " N. 86° 00′ W.
3. " " 83° 00′ "
4. " " 56° 30′ E.
5. " S. 45° 30′ "

EXPLANATION.—The Att. of the first B. is W. and the second B. is N. W., therefore, the Att. is added to it; the second Diff. being E. and less than the first, therefore, the Rem. should read W., consequently, be added to the next B., because it is a N. W. B. and the Att. W.

The difference between the first and second Diff. and the third Diff. is added to the fifth B., it being a S.E. B.

NOTE 3.—The Att. of the first B. was first applied, because the E. and W. Att. on the third B. was alike, hence it reversed.

By a perusal of the preceding, it is presumed, it may be seen that the rule and directions are easy of application to practical purposes.

☞ The example here given, is one of actual experiment, taken by the author in one of the Western States.

EXAMPLE.

B. S.=N. 54° 30′ W.=E.;—R. B. N.=N. 50° 45′ W.=W.; Diff. and Att.=3° 45′ W.; D. 4 ch. 07 links.
" N.=" 82° 15′ " =W.;— " S. =" 83° 30′ " =E.; " " " =1° 15′ E.; D. 5 " 16 "
" " =" 80° 30′ " =" ;— " " =" 80° 30′ " =" ; " " " =0° 00′ D. 8 " 38 "
" " =" 59° 00′ " =E.;— " " =" 60° 00′ " =W.; " " " =1° 00′ W.; D. 8 " 61 "
" S.=" 42° 00′ " =" ;— " N.=" 45° 30′ " =" ; " " " =3° 30′ E.; D. 4 " 91 "

POSITION.

Double Position.—The rule is founded on the supposition, that the first error is to the second as the difference between the true and first supposed number is to the difference between the true and second supposed number. When this is not the case, the exact answer to the questions cannot be found by the

RULE.

1. Take any two convenient numbers, and proceed with each according to the conditions of the question. Find how much the results are different from the result in the question. Multiply the first supposition by the second error, and the second supposition by the first error.

The difference between the result and the given sum is the error.

2. If the errors are alike—that is, both more or both less than the given sum, divide the difference of the products by the difference of the errors.

3. If the errors are unlike—that is, one larger and one smaller than the given sum, divide the sum of the products by the sum of the errors.

Remarks.—Questions that may be solved by *Single Position*, are better solved by analysis. Double Position is given here only for the gratification of the curious. The student may be tasked to perform the following questions by analysis.

EXAMPLES.

1. A and B have sheets of paper. A said to B, if I had two sheets more, I should have twice and half as much as you. B said to A, if I had 4 sheets more, I should have half as much as you. Required, the number of sheets that each had.

* Suppose 10, then 10+4=14.
Suppose 30, then 30+2=32.
Then, 32—14=18, *first error.*
Suppose 16, then 16+4=20. Suppose 40, then 40+2=42.
Then, 42—20=22, *second error.*
And, 40×18,—30×22=80. Also, 22—18=4.
Then, 80÷4=20 sheets, B had. And 20+4,×2=48 sheets, A had.

2. A, B and C have among them 135 guineas. A's plus B's are to B's plus C's as 5 to 7; C's minus B's to C's plus B's as 1 to 7. How many had each?

* Suppose A's + B's 50; then, as 5 : 50 : : 7 : 70.
And 70÷7=10, and 70—10,÷2=30, B's.
And 30+10=40, C's.
30—10=20, A's; 20+30+40=90, A's, B's and C's.
Then, as 90 : 135 : : 20 : 30, A's part.
And, as 90 : 135 : : 30 : 45, B's part.
Also, as 90 : 135 : : 40 : 60, C's part.

3. What fraction is that, to the numerator of which, if 1 be added, the value will be $\frac{1}{3}$; but if 1 be added to the denominator, its value will be $\frac{1}{4}$?

* Suppose $\frac{2}{9}$, then $\frac{2}{9}+{}^1=\frac{3}{9}=\frac{1}{3}$, but $\frac{2}{9}+{}_1=\frac{2}{10}$, which should be $\frac{2}{8}$ to be $\frac{1}{4}$; hence the error 2, too much; thus, 2—, that is, marked 2—, to signify 2 *more*. Had the error been 2 too little, it would have been marked 2+, that is 2 *less;* for the answers to arithmetical questions, if too small, or too large, are thus expressed.

Suppose $\frac{1}{6}$, then $\frac{1}{6}+{}^1=\frac{2}{6}=\frac{1}{3}$, but $\frac{1}{6}+{}_1=\frac{1}{7}$, not $\frac{1}{4}$, therefore, the error is 3, too much.

Then, $\left\{\begin{array}{l} 2\times3,-1\times2=\ 4,\div3-2=\frac{4}{15}, \text{ numerator.} \\ 9\times3,-6\times2=15,\div3-2=15, \text{ denominator.} \end{array}\right\}$ Ans.

4. A, B, C and D found a purse of dollars, and each of them took a number at a venture, afterwards by comparing their shares, they found that if A took 25 from B, his number would be equal to what B had left, and if B took 30 from C, his number would be three times what C had left, and if C took 40 from D, his number would be double to what D had left, and if D took 50 from A, his number would be three times as much as A had left, and $5 over. What number had each?

* 1. Suppose A had 94, then 94+25+25+30,÷3=58.
58+30+40=128,÷2=64,+40=104,+50=154.
Then, 94—50=44,×3=132; 154—132=22; 22—5=17, first error.

2. Suppose A had 106, then 106+25+25+30,÷3=62.
62+30+40=132,÷2=66,+40=106,+50=156.
Then, 106—50=56,×3=168; 168—156=12; 12+5=17, second error.

Then, 106×17,+94×17,÷17+17=34.
And 3400÷34=100 dollars, A had.
Hence, B had 150, C 90, and D 105.

5. A said to B and C, give me half of your money, and I shall have \$100; B said to A and C, give me $\frac{1}{3}$ of yours, and I shall have \$100; and C said to A and B, give me $\frac{1}{4}$ of yours, and I shall have \$100. How much money had each man?

* For the solution of the above question, see the margin. In the operation, consider 2+1+1, 1+3+1, and 1+1+4, also as multiplied and subtracted in each part of each operation, producing 51,34, and 51 as the divisors, or in other words, the errors; the dividends being the results produced from proceeding according to the conditions of the question.

6. A said to B, C and D, give me $\frac{1}{2}$ of your money, and I shall have \$200; B said to A, C and D, give me $\frac{1}{3}$ of yours, and I shall have \$200; C said to A, B and D, give me $\frac{1}{4}$ of yours, and I shall have \$200; D said to A, B and C, give me $\frac{1}{5}$ of yours, and I shall have \$200? How much money had each man?

* 1. Suppose A had \$1; then B, C and D together had \$398; if $\frac{1}{2}$ their's, added to A's \$1 made \$200. And of the \$398, B must have \$100$\frac{1}{2}$, C \$133$\frac{2}{3}$, and D the remainder, \$163$\frac{5}{6}$. These proportions will give \$200 for A, and the same for B and C each; but A's \$1+B's \$100$\frac{1}{2}$+C's \$133$\frac{2}{3}$=235$\frac{1}{6}$, to $\frac{1}{5}$ of which add D's share, \$163$\frac{5}{6}$, and the sum is \$210$\frac{26}{30}$, yet it should have been \$200; therefore, the error is 10$\frac{13}{15}$, *too great.*

2. Suppose A had \$2, then B, C and D together had \$396, and all four of them had \$398, and of the \$396, B must have \$101, C \$134, and D the remainder, \$161. These numbers will give A, B and C's shares in the right proportion; but $\frac{1}{5}$ of them added to what is left for D, \$161, makes \$208$\frac{2}{5}$ for D, yet it should have been \$200; hence, the second error is 8$\frac{2}{5}$, *too great.*

A+B+C=A's.

2+1+1=200×3−1+3+1=300,×11−1+1+4=400×3−1+3+1=300,×2=51)1500=\$29$\frac{7}{17}$, A's.

A+B+C=B's.

1+3+1=300×2−2+1+1=200,×7−1+1+4=400×2−2+1+1=200=34)2200=\$64$\frac{12}{17}$, B's.

A+B+C=C's.

1+1+4=400×3−1+3+1=300,×5−2+1+1=200×3−1+3+1=300,×2=51)3900=\$76$\frac{8}{17}$, C's.

The suppositions and errors will then stand as follows :—

For A.		For B.		For C.		For D.	
1	$10\frac{13}{15}-$	$101\frac{1}{2}$	$10\frac{13}{15}-$	$133\frac{2}{3}$	$10\frac{13}{15}-$	$163\frac{5}{6}$	$10\frac{13}{15}-$
2	$8\frac{2}{5}-$	101	$8\frac{2}{5}-$	134	$8\frac{2}{5}-$	161	$8\frac{2}{5}-$

As the same errors are used in finding every share, it will be most convenient to reduce them to 15ths, before multiplying.

3. It now remains to show how to find B's and C's proportions of the first and second supposed sums, $399, and $398, for after being found, the most difficult part of the work is performed, A's being *assumed*, and D's being found by subtracting A's, B's and C's from each whole supposed number, viz., $399, and $398.

4. *To find B's part of the* $399. Suppose $51; from $399-51, \div 3 = 116$, $\frac{1}{3}$ of A, C and D's, which plus $51 = 167$; then $200-167=33$, first error.

Suppose $90; from $399-90, \div 3 = 103$, $\frac{1}{3}$ of A, C and D's, which plus $90=193$; then, $200-193=7$, second error.

These suppositions and errors give $$101\frac{1}{2}$ for B.

5. *To find C's part of the* $399. Suppose $83.

Then, from $399-83, \div 4 = 79$, $\frac{1}{4}$ of A, B and D's, which plus $83=162$; then, $200-162=38$, first error.

Suppose $103; from $399-103, \div 4 = 74$, $\frac{1}{4}$ of A, B and D's, which plus $103=177$; then, $200-177=23$, second error.

These suppositions and errors give $$133\frac{2}{3}$ for C.

6. *To find D's share of the* $399. Subtract A, B and C's from the whole.

Proceed as in the last two cases to find all the proportions of the second supposed number, $398.

Ans. A had $$5\frac{15}{37}$, B $$102\frac{26}{37}$, C $$135\frac{5}{37}$, and D $$151\frac{13}{37}$.

7. At a certain time between two and three o'clock, the minute hand of the clock was between three and four. Within an hour after, the hour hand and minute hand had exactly changed places with each other. What was the precise time, when the hands were in the first position?

* 1. Suppose the time to be 16 minutes past 2 o'clock, the hour hand must have passed $\frac{16}{60}$ of the distance from 2 o'clock to 3 o'clock; and if the minute hand was in the place of the hour hand, it would be 11m. 20 sec. from 12 o'clock; and if the hour hand was in the place of the minute hand, it would be 12 minutes past 3 o'clock.

The difference between 12 m. and 11 m. 20 sec, is 40 seconds.

Therefore, let 40 sec. be the first error.

2. Suppose the time to be 18 minutes past two o'clock; the hour hand, at that time, has passed $\frac{18}{60}$ of the distance from 2 o'clock to 3 o'clock; and if the minute hand was in the place of the hour hand, it would be 11 m. 20 sec. from 12 o'clock. And if the hour hand was in the place of the minute hand, the time would be 36 minutes past 3 o'clock.

The difference between 36 m. and 11 m. 30 sec. is 24 m. 30 sec., being 1470 seconds.

Let this be the second error.

3. $1470\times16,-40\times18,\div1470-40=15$ m. $56\frac{92}{143}$ seconds past 2 o'clock, Ans.

MISCELLANIES.

Remarks.—Besides the remarks on Proportionals and Percentage Proportionals, it may not be amiss to observe, that, the same said of them is also applicable to this division and the Promiscuous Questions in Mensuration, and it may be said of these four divisions, that, the operations of their questions, though the common method of solving such, are to be considered but the outlines or notes of their thorough analysis, of which the questions are the texts acting as *exercises;* consequently, pupils and the teacher should go hand in hand in giving the analysis; and from this course, one may judge the result.

EXAMPLES.

1. A lady purchased a piece of silk frocking, at 80 cents per yard, and lining for it, at 30 cents per yard; the frock and lining contained 15 yards, and the price of the whole was $7,00. How many yards were there of each?

* $7÷15=46⅔ cents, average price per yard.

$80-46\frac{2}{3}=33\frac{1}{3}$; $46\frac{2}{3}-30=16\frac{2}{3}$; hence, it is evident, that the quantity of lining will be to that of the silk as $33\frac{1}{3}$ to $16\frac{2}{3}$, that is, the quantity of lining will be double the quantity of silk. Hence, 10 yards of lining, and 5 yards of silk.

2. A and B bought a quantity of calico for $30,59. A paid 15 cents per yard for his, and the price per yard of B's was equal to $\frac{1}{7}$ of the whole number of yards. Required, the price of B's per yard.

*$\sqrt{(3059\times2\div7+15\div14)}=61$; then, $61-15,\div2=23$ cents, Ans.

Questions of this nature admit of an infinite number of answers, one as correct as the other.

3. A party of lively young gentlemen and ladies, going into the country on a tour of pleasure, had a bill of $24,99 to pay, part of which the funny females insisted on discharging; hence, it was agreed that each gentleman should pay $1,17 cents of this expense, and each lady 34 cents. What number of each sex was there?

* Rule.—Divide the amount to be paid by one of the respective shares, or some multiple of it, till the remainder becomes divisible by the other. Ans., 17 gentlemen, and 15 ladies.

4. A man was hired 50 days on these conditions that; for every day he wrought he was to have 75 cents, and for every day he was idle he was to forfeit 25 cents. How many days was he idle, if he received $27.50 at the end of the time?

*Had he worked every day, his wages would have been $37.50, $10 more than he received; but every day he was idle lessened the $37.50 just 75 cents plus 25 cents, $1.00; therefore, he was idle 10 days.

5. A and B have the same income. A saves an eighth of his; B spending $30 yearly more than A, finds himself $40 in debt at the end of 8 years. Required their income and what each spends per annum.

*$40÷8=$5 yearly, more than his income, $30—5=$25, what A saves yearly; hence, $25=$\frac{1}{8}$ of either's income; consequently $200 their income.

6. The head of a fish weighs 4lbs., its tail weighs as much as its head and half of its body, and its body weighs as much as head and tail. Required the weight of the fish.

*The head and tail weigh 8lbs. and $\frac{1}{2}$ of the body, and as the body weighs as much as the head and tail, it is evident that 8lbs. is $\frac{1}{2}$ of the required weight.

Hence, its tail weighs 4 lbs. plus 8lbs. equal 12lbs.

Lastly, $4+12+16=32$lbs., weight of the fish, Ans.

7. A man when he was married was 3 times as old as his wife; 15 years after, he was but twice the age of his lady. At what age was each married?

*When they were married her age was 1 year to his 3 years; in 15 years his age was 2 to her 1, that is, 15 years doubled her age, and his was $\frac{4}{3}$ what it was; hence, he was 45 and the lady 15 years of age.

8. In a mixture of wine and cider, $\frac{1}{2}$ of the whole plus 25 gallons is wine, and $\frac{1}{3}$ part less 5 gallons is cider. How many gallons of each kind in this mixture?

* $\frac{1}{2}+\frac{1}{3}=\frac{10}{12}$ in all. Therefore, $25-5=20=\frac{1}{6}$ of the required quantity; hence, 120 gallons in all. Consequently, 85 gallons is wine, and 35 gallons is cider.

9. A man bought some lemons at 2 cents each, and $\frac{3}{4}$ as many at 3 cents each, and then sold them all at the rate of 5 cents for 2, and thus gained 25 cents. How many lemons did he buy?

*He bought 4 at 2 cents each, as often as he bought 3 at 3 cents each, therefore he gave 17 cents for every 7 lemons, at $2\frac{3}{7}$ cents each, but sold them at $2\frac{1}{2}$ cents each.

Then, $2\frac{1}{2}-2\frac{3}{7}=\frac{1}{14}$ of a cent gained on one, consequently 25 cents gained on 350 lemons, Ans.

10. The stock of a cotton manufactory is divided into 32 shares, and owned equally by 8 persons, A, B, C, &c. A sells 3 of his shares to a ninth person, who thus becomes a member of the company, and B sells 2 of his shares to the company, who pay for them from the common stock. After this, what proportion of the whole stock does A own?

*Each one owns $\frac{4}{32}$ of the whole, and A reserved but $\frac{1}{32}$ of his, and 2 of B's shares being taken up by the company, hence only 30 shares in all, consequently A owns $\frac{1}{30}$ of the whole.

11. A lady has two silver cups of unequal weight with but one cover. The first cup weighs 12oz. If the first cup be covered, it will weigh twice as much as the second; but if the second cup be covered, it will weigh three times as much as the first. What is the weight of the cover and of the second cup?

*First cup, oz. 12×3, second cup and cover, viz. 26oz.; hence, both cups and cover 48oz., its $\frac{2}{3}=32$oz., first cup and cover, for either cup covered are to each other as 2 to 3; then, $48-32=16$oz., second cup, and $48,-16+12=28$ oz., cover.

12. A general disposing his army into a square, found he had 231 over and above; but increasing each side with one soldier, he wanted 44 to fill up the square. Of how many men did his army consist?

*$231+44+1,\div2=138$; then, $138\times138,-44=19000$ men, Ans.

The 1 is added because the soldier standing in the corner of the square is counted twice.

13. A military officer drew up his soldiers in rank and file, having the number in rank and file equal; on being reinforced with three times his first number of men, he placed them all in the same form, and then the number in rank and file was double what it was at first; he was again reinforced with three times his *whole number* of men, and after placing the whole in the same form as before, his number in rank and file was 40 men each. How many men had he at first?

*His first number call 1; his first reinforcement is 3, and second 12; therefore, $40\times40,\div1+3+12=100$ men, number at first, Ans.

Or, $\frac{2}{2}\times3,+\frac{2}{2}=\frac{8}{2},\times3,+\frac{8}{2}=\frac{32}{2}$.

Then, $40\times40\times2,\div32=100$ men, Ans.

14. A fellow said that when he counted his plums two by two, three by three, four by four, five by five, and six by six, there was still an odd one; but when he counted them seven by seven they came out even. How many had he?

*Thus, $2\times3\times4\times5\times6+1=721$, Ans.

15. A paid \$100 for 100 animals, consisting of oxen, sheep, and geese, paying \$10 for an ox, \$1 for a sheep, and a shilling for a goose, respectively. How many of each did he buy?

*First, $6\left\{\begin{array}{l}60\\1\end{array}\right.$ 5 oxen, 54 geese, | Then, $100-54+5=41$ sheep.

16. How much wine at 4s. 6d. and at 5s. per gallon must be mixed with 6 gallons at 4s. and 6 gallons at 3s. per gallon, that the mixture may be worth 4s. 6d. per gallon?

*Limited, { 6gls. at 4s.=24 }
Simples, { 6 " at 3s.=18 }

12 . . . . .)42=3s. 6d., rate of the limited simples.

Then, 4s. 4d.=52 $\left\{\begin{array}{l}42\\54\\60\end{array}\right.$ | 2+8, 10, 10 | 10, 10, 10

And, as 10 : $\left\{\begin{array}{l}10\\10\end{array}\right\}$: : 12 : $\left\{\begin{array}{l}\text{12gls. at 4s. 6d.}\\\text{12gls. at 5s.}\end{array}\right\}$ per gallon.

17. What will 19 tons 19cwt. 3qrs. $27\frac{1}{2}$lbs. of iron cost at £19 19s. 11d. 3qrs. per ton ?

	*£19	19s.	$11\frac{3}{4}$d., price of one ton.
			18=18 ton.
	359	19	$7\frac{1}{2}$, price of 18 ton.
10cwt.=$\frac{1}{2}$	19	19	$11\frac{3}{4}$, price of the 19th ton.
5cwt.=$\frac{1}{2}$	9	19	$11\frac{7}{8}$,
4cwt.=$\frac{4}{5}$	4	19	$11\frac{15}{16}$
2qrs. =$\frac{1}{8}$	3	19	$11\frac{19}{20}$
1qr. =$\frac{1}{2}$		9	$11\frac{153}{160}$
14lbs. =$\frac{1}{2}$		4	$11\frac{313}{320}$
7lbs. =$\frac{1}{2}$		2	$5\frac{633}{640}$
$3\frac{1}{2}$ " =$\frac{1}{2}$		1	$2\frac{1273}{1280}$
2lbs. =$\frac{4}{7}$			$7\frac{1273}{2560}$
1lb. =$\frac{1}{2}$			$4\frac{1273}{4480}$
			$2\frac{1273}{8960}$
Ans.	£399	19s.	$5\frac{16641}{17920}$d.

NOTE.—The sum of the several fractions is $8\frac{16641}{67920}$d.

18. C owes D $1400, to be paid in 3 months; but D being in want of money, C pays him at the expiration of 2 months, $1000; how much longer than 3 months ought C, in equity, to defer the payment of the remainder ?

*Interest of $1000 for one month is $5; hence, the $400 should be kept till it amounts $405, and $400 will gain $24 interest in 12 months.

Therefore, as $24 : 12m. : : $5 : $2\frac{1}{2}$ months, Ans.

REMARK.—*Strict justice*, in equation of payments, demands that *interest* should be paid on all sums, from the time they become due, until the time of payment; and the *present worth* of all sums, paid *before* they are due. But a different, though incorrect, rule is generally adopted for this purpose, as also is the case with a great portion of the rules in a common arithmetic.

19. If Paris, in France, be in 2° 20′ east longitude from Greenwich, in England, and Hallowell in 69° 42′ west longitude from Greenwich; when it is noon at Paris, what time of day is it at Hallowell ?

*If 1° : 4m. : : 69° 42′+2° 20′ : 4h. 48sec. 8‴; then, 12h. less 4h. 48sec. 8‴ gives 7h. 11m. 52s. in the morning, Ans.

20. If a meteor appears so high in the heavens as to be visible at Boston, 71° 3′, at the city of Washington, 77° 43′, and at

the Sandwich Islands, 155° W. longitude, and that its appearance at the city of Washington be at 7 minutes past 9 o'clock in the evening; what will be the hour and minute of its appearance at Boston and at the Sandwich Islands?

*If at the place *easterly* of another the Sun appears sooner than at the place *westerly*, it follows, that when it is 12 o'clock at the place easterly, it is but 11 o'clock at a place 15° westerly, since the earth's motion is easterly 15° per hour. Therefore, at Boston it is *later* in the evening than at Washington, and consequently it is *earlier* in the evening at the Sandwich Islands, than 7 minutes past 9 o'clock. And the difference of longitude of either two of the places, reduced to time, is their difference of time, which difference add to the given time, if the longitude of the required time be east, but subtract it, if it be west.

Then, 77° 43′—71° 3′,×4=27m. 40s.,+9h. 7m., which is, 9h. 34m. 40s., time in the evening at Boston.

Again, 155°—77° 43′,×4=5h. 9m. 8s., which taken from 9h. 7m.=3h. 57m. 52s. time in the afternoon at the Sandwich Islands.

21. A family of 10 persons took a large house for ½ of a year, for which they agreed to pay $500 for that time. At the end of 14 weeks, they took in 4 new lodgers; and after 3 weeks, 4 more; and so on at the end of every 3 weeks, during the term, they took in 4 more. How much rent must one of each class pay?

*At the beginning of the last three weeks, the family amounted to 26 persons, it being but 10 at first, for the 14 first weeks, increasing by 4 persons every 3 weeks; therefore, first find the rent of the house for 14 weeks, and divide it among the first 10 lodgers; then find the rent for 3 weeks, and divide it first among 14 lodgers, then 18, &c., to the end of the time. Or,

Thus, find a middle term for each of the several proportional statements, by dividing the rent, severally, by 26, 22, 18, 14, and the first 10; then use 26 weeks, half a year, for the first term in each statement, and 3, 3, 3, 3, and 14 weeks, severally, for the third terms.

Thus, 26 : 500 : : 3 : 57.692\frac{52}{169}$, ÷26=$2.218$\frac{158}{169}$, each of 5th class should pay.

Then, 26 : 500,÷22 : : 3 : 2.622\frac{198}{286}$,+$2.218$\frac{158}{169}$, one of 4th class.

And, 26 : 500 : :,÷18 : : 3 : difference between one of

4th and one 3d class, which plus one of 4th class gives 8.046\frac{2458}{5577}$, one of 3d class.

The same of the remaining classes, which gives the 2d class 12.167\frac{12487}{39039}$ each, and the 1st class 39.090\frac{15490}{39039}$ each to pay.

22. If 12 oxen graze off $3\frac{1}{3}$ acres of grass in 4 weeks; and 21 oxen graze off 10 acres in 9 weeks; how many oxen would it require to graze off 24 acres in 18 weeks; the grass being at first equal on every acre, and growing uniformly?

*If $\left.\begin{matrix} 12\text{ox.} : 4\text{W.} : : 3\frac{1}{3}\text{A.} \\ 18\text{W.} : 10\text{A.} \end{matrix}\right\}$: 8 oxen to graze off 10 acres in 18 weeks, supposing the grass *not to grow.*

By the question, 21 oxen graze off 10 acres of grass in 9 weeks, on account of its growing; whereas, by the work, if the grass had not grown, 16 oxen would have grazed it off in the same time, because 8 oxen 18 weeks, is the same as 16 oxen 9 weeks. Therefore, it is evident, the *growth* of grass on 10 acres, for the excess of 9 weeks more than 4 weeks, will feed the excess of 21 oxen more than 16 oxen for 9 weeks, that is, 5 oxen for 9 weeks; or, which is the same thing, 2.5 oxen 18 weeks.

Again, if 9W. less 4W. : 2.5ox. : : 18W. less 4W. : 7 oxen.

Hence, the *growth* of the grass on 10 acres during 18 weeks is grazed off by 7 oxen, and 8 oxen will graze off the grass at first standing on 10 acres; consequently, 15 oxen would graze off 10 acres, growth and all, in 18 weeks.

Lastly, as 10A. : 15ox. : : 24A. : 36 oxen, Ans.

N. B.—The preceding question, it is believed, was first proposed by Sir Isaac Newton; it may be found in Emerson's third part on Arithmetic, the quantities being such as to render a solution difficult on account of the fractions which would occur, that is its greatest merit in his work, if obscuring such questions deserve merit. A *prize*, it is said, of $50 was awarded for its most "*lucid analytical solution,*" presented by James Robinson, Principal of the Department of Arithmetic, Bowdoin School, Boston.

23. A, in a scuffle, seized on $\frac{2}{3}$ of a parcel of sugar-plums, B caught $\frac{3}{8}$ of them out of his hands, and C laid hold on $\frac{3}{10}$ more; D ran off with $\frac{5}{7}$ of what A had left, and the rest E afterwards secured slyly for himself. Then A and C jointly fell upon B, who, in the conflict, let fall $\frac{1}{2}$ he had, which were equally picked up by D and E.

B then kicked down C's hat, and at it they went anew for what it contained; of which A got $\frac{1}{4}$; B $\frac{1}{3}$; D $\frac{2}{7}$; and C and E equal shares of what was left of that stock.

D then struck $\frac{3}{4}$ of what A and B last acquired, out of their hands; they with difficulty recovered $\frac{5}{8}$ of it in equal shares again, but the other three carried off $\frac{1}{8}$ apiece of the same. Upon this, they called a truce, and agreed that the third left by A at first should be equally divided among them. How much of the prize, after this distribution, had each of the competitors?

* I. A got $\frac{2}{3}$, and B got $\frac{3}{8}$ of $\frac{2}{3}=\frac{6}{24}=\frac{1}{4}$.

Then C got $\frac{3}{10}$ of $\frac{2}{3}=\frac{6}{30}=\frac{1}{5}$.

Then, $\frac{1}{4}+\frac{1}{5}=\frac{9}{20}$, and $\frac{2}{3}-\frac{9}{20}=\frac{13}{60}$, what A had left. D ran off with $\frac{6}{7}$ of $\frac{13}{60}=\frac{78}{420}$, and E secured $\frac{1}{7}$ of $\frac{13}{60}=\frac{13}{420}$; consequently, A had none left. Hence, at the end of the first heat, B had $\frac{1}{4}$; C, $\frac{1}{5}$; D, $\frac{78}{420}$ and E, $\frac{13}{420}$.

II. B let fall $\frac{1}{2}$ of $\frac{1}{4}=\frac{1}{8}$, and had $\frac{1}{8}$ left. D picked up $\frac{1}{2}$ of $\frac{1}{8}=\frac{1}{16}$, and E the same quantity.

Then D had $\frac{78}{420}+\frac{1}{16}=\frac{417}{1680}=\frac{139}{560}$.

And E had $\frac{13}{420}+\frac{1}{16}=\frac{157}{1680}$.

Therefore at the end of the second heat, their shares stood thus: B, $\frac{1}{8}$; C, $\frac{1}{5}$; D, $\frac{139}{560}$; E, $\frac{157}{1680}$.

III. A got $\frac{1}{4}$ of $\frac{1}{5}=\frac{1}{20}$; B got $\frac{1}{3}$ of $\frac{1}{5}=\frac{1}{15}$; and D got $\frac{2}{7}$ of $\frac{1}{5}=\frac{2}{35}$. Theirsum is $\frac{1}{20}+\frac{1}{15}+\frac{2}{35}=\frac{73}{420}$, and $\frac{1}{5}-\frac{73}{420}=\frac{11}{420}$, what C had left. Then C took $\frac{1}{2}$ of $\frac{11}{420}=\frac{11}{840}$, and E took the same. B then had $\frac{1}{8}+\frac{1}{15}=\frac{23}{120}$; D had $\frac{139}{560}+\frac{2}{35}=\frac{171}{560}$.

And E had $\frac{157}{1680}+\frac{11}{840}=\frac{179}{1680}$. Consequently, at the end of this engagement, their shares were as follows;

A, $\frac{1}{20}$; B, $\frac{23}{120}$; C, $\frac{11}{840}$; D, $\frac{171}{560}$; and E, $\frac{179}{1680}$.

IV. $\frac{1}{20}+\frac{1}{15}=\frac{35}{300}=\frac{7}{60}$, what A and B last acquired. D struck out of their hands $\frac{3}{4}$ of $\frac{7}{60}=\frac{21}{240}=\frac{7}{80}$.

Then A had $\frac{1}{4}$ of $\frac{1}{20}=\frac{1}{80}$ left, and B had $\frac{1}{4}$ of $\frac{1}{15}=\frac{1}{60}$ left.

They recovered, in equal shares, $\frac{5}{8}$ of $\frac{7}{80}=\frac{35}{640}=\frac{7}{128}$; consequently, A took $\frac{1}{2}$ of $\frac{7}{128}=\frac{7}{256}$, and B took as much, that is, $\frac{7}{256}$.

Then A had $\frac{1}{80}+\frac{7}{256}=\frac{51}{1280}$, and B had $\frac{1}{60}+\frac{7}{256}+\frac{1}{8}=\frac{649}{3840}$.

Then C, D and E got each $\frac{1}{8}$ of $\frac{7}{80}=\frac{7}{640}$.

Consequently C had $\frac{11}{840}+\frac{7}{640}=\frac{323}{13440}$.

And E, $\frac{179}{1680}+\frac{7}{640}=\frac{1579}{13440}$.

Hence their shares after the fourth heat were thus:

A, $\frac{51}{1280}$; B, $\frac{649}{3840}$; D, $\frac{1417}{4480}$; E, $\frac{1579}{13440}$.

V. After the truce, each was to have $\frac{1}{5}$ of $\frac{1}{3}=\frac{1}{15}$; consequently, their shares stood as follows:

A's share was $\frac{51}{1280}+\frac{1}{15}=\frac{409}{3840}=\frac{2863}{26880}$.
B's share was $\frac{649}{3840}+\frac{1}{15}=\frac{905}{3840}=\frac{6335}{26880}$.
C's share was $\frac{323}{13440}+\frac{1}{15}=\frac{1219}{13440}=\frac{2438}{26880}$.
D's share was $\frac{1417}{4480}+\frac{1}{15}=\frac{5147}{13440}=\frac{10294}{26880}$.
E's share was $\frac{1579}{13440}+\frac{1}{15}=\frac{2475}{13440}=\frac{4950}{26880}$.

The sum of the numerators of the last fractions, is equal to the common denominator, and, consequently, is the whole number of sugar-plums. Hence the numerators express their respective shares. Ans. A had 2863; B, 6335; C, 2438; D, 10294; E, 4950.

24. A Problem.—*Knowing the price for which a thing is sold, in which there is as much gained per cent. as the article cost, to find the original cost.*

Rule.—Multiply the price or sum sold for by 100, add the square of 50 to the product; from the square root of this sum subtract 50, and the remainder will be the first cost.

Reason.—As the rate per cent. is the same as the principal, if the first cost or principal be multiplied by the rate, it is the same as squaring the principal, which square or product divided by 100, must give the interest.

EXAMPLES.

1. Sold a piece of cloth for £24, and gained as much per cent. as the cloth cost me; what was the price or gain per cent.?

*$\sqrt{(24\times100,+50\times50)}=70,-50=$£20, its original cost or gain per cent.

2. Sold a piece of cloth for £56, and gained as much per cent. as the cloth cost me; what was the gain per cent.?

*$\sqrt{(56\times100,+50\times50)}=90,-50=40$, its cost or gain per cent.; then, $40\times40,\div100=16$, the whole gain.

Hence, if 40 : 16 : : 100 : 40 per cent., *gain*.

Remark.—The preceding rule is founded on the principles of algebra, as will appear to those acquainted with that science, from the algebraic solution here given without letters.

SUNDRY TABLES.

I. OF CIRCLES.

If the diameter of any circle

be $\left\{\begin{array}{l}\text{multiplied}\\ \text{divided}\end{array}\right\}$ by $\left\{\begin{array}{r}3.14159\\ .31831\end{array}\right\}$ $\begin{array}{l}\text{the product}\\ \text{the quotient}\end{array}\Bigg\}$ is the circumference; Or, if it

be $\left\{\begin{array}{l}\text{multiplied}\\ \text{divided}\end{array}\right\}$ by $\left\{\begin{array}{r}.886227\\ 1.128370\end{array}\right\}$ $\begin{array}{l}\text{the product}\\ \text{the quotient}\end{array}\Bigg\}$ is the side of an equal square; Or, if it

be $\left\{\begin{array}{l}\text{multiplied}\\ \text{divided}\end{array}\right\}$ by $\left\{\begin{array}{l}.866024\\ .1547\end{array}\right\}$ $\begin{array}{l}\text{the product}\\ \text{the quotient}\end{array}\Bigg\}$ is the side of an equilaterial triangle inscribed; Or, if it

be $\left\{\begin{array}{l}\text{multiplied}\\ \text{divided}\end{array}\right\}$ by $\left\{\begin{array}{r}.707016\\ 1.414213\end{array}\right\}$ $\begin{array}{l}\text{the product}\\ \text{the quotient}\end{array}\Bigg\}$ is the side of a square inscribed.

II. *Thermometer.*

Deg.	Feet.
32	86.86
35	87.49
40	88.54
45	89.60
50	90.66
55	91.72
60	92.77
65	93.82
70	94.88
75	95.93
80	96.99

III. *Showing the amount of* $1.

Rate.	Half Yearly.	Quarterly.
3	1.007445	1.011181
3.5	1.008675	1.013031
4	1.009902	1.014877
4.5	1.011126	1.016720
5	1.012348	1.018559
5.5	1.013567	1.020395
6	1.014781	1.022257
6.5	1.015993	1.024055
7	1.017204	1.025880

N.B.—The above may be used as multipliers, in obtaining the amount of any sum, at those rates, and for half yearly or quarterly payments.

IV. *Of Superficies, viz.:* POLYGONS.

A figure having ... equal sides and angles is a

3	Trigon, . . .	.433013
4	Tetragon, . .	1.000000
5	Pentagon, . .	1.720477
6	Hexagon, . .	2.589076
7	Heptagon, . .	3.633959
8	Octagon, . . .	4.828427
9	Enneagon, . .	6.181827
10	Decagon, . .	7.694209
11	Endecagon, .	8.514250
12	Dodecagon, .	9.330125

NOTE.—If the square of a side of either figure described in this Table, be multiplied by the number at the right of it, the product will be its true area.

V. *Of Solids and their Superficies, viz.:* Polyedrons: Being five solids contained under equal regular sides, termed *the five regular bodies.*

Names of the Bodies.	Solidity.	Superficies.
Tetraedron,	0.11785	1.73205
Hexaedron,	1.	6.
Octaedron,	0.4714	3.464
Eicosiedron,	2.181695	8.66025
Dodecaedron,	7.663119	20.6457

Note.—All like solids being in proportion to one another as the cubes of their homologous sides, the solid content of any of the bodies named in Table V., may be found by multiplying the cubes of their sides by the numbers in the column under *Solidity;* and their superficies, by multiplying the squares of their sides into the numbers under *Superficies.*

The Tetraedron—Is a triangular pyramid of four equal faces, the side of whose base is equal to the slant height of the pyramid, from the angles to the vertex.

The Octaedron—Consists of two quadrangular pyramids, of equal bases, joined together, the sides of whose bases are equal to the given sides of the triangles, under which it is contained.

The Dodecaedron—Consists of twelve pentagonal pyramids, of equal bases and altitude, whose vertices meet in the centre of the dodecaedron.

The Eicosiedron—Consists of twenty equal triangular pyramids, whose vertices all meet in the centre.

VI. *Showing the Proportions which the following Solids have to the Cube and Cylinder, having the same Base and Altitude.*

	Solid Inches
1. A *Cube*, whose side is 12 inches, contains	1728
2. A *Prism*, having an equilateral triangle, whose side is 12 inches for its Base, and its Altitude 12 inches, contains - - - -	784,24
3. A *Square Pyramid*, whose height and the side of its Base are each 12 inches, is $\frac{1}{3}$ of the above cube, and therefore contains - -	576
4. A *Triangular Pyramid*, whose height and side of its Triangular Base are each 12 inches, is near $\frac{1}{7}$ of the cube, and contains - -	249,413
5. A *Cylinder*, whose diameter and height are each 12 inches, is $\frac{11}{14}$ of the above cube, and contains	1357,17
6. A *Sphere* or *Globe*, whose axis or diameter is 12 inches, equal to the side of the cube, is $\frac{11}{21}$ of it, and contains - - - - - -	904,78
7. A *Cone*, whose base and altitude are each 12 inches, equal to the side of the cube, is $\frac{5}{19}$ of it, and contains - - - -	452,38829
8. A *Parabolic Conoid*, whose diameter at the base, and height are each 12 inches, being $\frac{1}{2}$ its circumscribing cylinder, contains - -	678,583
9. A *Hyperbolic Conoid*, whose height and diameter at the base, are each 12 inches, is $\frac{5}{12}$ of its circumscribing cylinder, and contains -	565,49
10. A *Parabolic Spindle*, whose height and middle diameter are each 12 inches, is $\frac{8}{15}$ of its circumscribing cylinder, and contains - -	723,824

Hence arises a different method of finding their contents.

General Rule.—If the base of the solid whose content you would find, be rectilinear, consider it as a *Parallelopipedon;* if curved, as a *Cylinder;* and find the content accordingly. Then take such a part of the content, thus found, as is specified in the preceding Table, which, if the parts be taken in inches, will be the solid content of the given figure, in inches, which, divided by 1728, will give the cubic feet.

VII. *Showing the Specific Gravities of several solid and fluid Bodies.*

METALS.		Bituminous coal, from	1,100
Platina, pure, -	19,500	to - - - - - -	1,300
Platina, hammered, -	21,500	WOODS, ETC.	
Gold, pure and cast, -	19,260	Lignum Vitae, - -	1,300
Gold, hammered, -	19,360	Ebony - - - - - -	1,200
Mercury, - - - - -	13,560	Hempen rope, or cable,	1,100
Lead, cast, - - -	11,350	Mahogany, - - - - -	1,000
Silver, pure and cast,	10,470	Boxwood, - - - - -	1,000
Silver, hammered, -	10,510	Shell bark Hickory,	1,000
Silver, standard, -	10,535	White Oak, heart, -	,930
Copper, cast, - -	8,790	Ash, - - - - - -	,800
Copper, hammered, -	8,890	Rock Maple, - - - -	,760
Brass, cast, - - -	8,400	White Pine, - - - -	,570
Brass, hammered, - -	8,500	Charcoal, - - - - -	,400
Iron, cast, - - - - -	7,210	Cork, - - - - - -	,240
Iron hammered, - -	7,790	LIQUIDS, ETC.	
Steel, - - - - - -	7,840	Sulphuric acid, - -	1,840
Tin, cast, - - - - -	7,300	Nitric acid, - - -	1,220
Zinc, cast, - - - - -	7,200	Sea water, - - - - -	1,030
STONES, EARTHS, ETC.		Cow's milk, - - - -	1,030
Granite, - - - - -	2,700	Pure fresh water, - -	1,000
Marble, - - - - -	2,700	Whale Oil, - - - -	,920
Slate, - - - - - -	2,700	Tallow, - - - - - -	,920
Glass, - - - - - -	2,600	Olive Oil, - - - - -	,910
Flint Stone, - - -	2,580	Proof spirits, - - -	,920
Paving Stone, - -	2,580	Alcohol, - - - - -	,840
Free Stone, - - -	2,500	GASES.	
Clay, - - - - - -	2,200	Oxygen gas, - - - -	,00134
Sand, - - - - - -	1,500	Carbonic acid gas, -	,00164
Anthracite coal, from	1,400	Common air, - -	,00122
to - - - - - -	2,000	Nitrogen gas, - - -	,00098
Brick, from 1,800 to -	2,000	Hydrogen gas, - -	,00008

REMARKS.—Omit the decimal point, and the numbers will be the Avoirdupois ounces in a cubic foot of each substance, which is termed its *Absolute Weight.*

The specific gravity of any solid, liquid, or gas, increases with the cold, and diminishes with the heat. Moreover, there is always some difference in the specific gravity of several varieties of the same substance.

ARITHMETIC ODDITIES.

1. $99\frac{9}{9}=4$ nines counting 100; and $11\frac{1}{1}=4$ ones counting 12.

2. $35+29=64,+8+7+1$, the 9 digits so placed as to count 80.

3. And $\left\{\begin{matrix}438\\951\\276\end{matrix}\right\}$ the 9 figures so placed, that any three figures in a right line makes just 15.

4. If a body of 40 lbs. weight be divided into 4 such parts, that one of which will weigh 1 lb., another 3 lbs., another 9 lbs.. and the other 27 lbs., those several parts may be so used as weights to a balance, as to weigh any number of pounds, from 1 lb. to 40 lbs.; the ratio being 3, if the body had been 121 lbs., and it had been required to find the least number of weights with the weight of each part, to weigh any number of pounds from 1 to 121 lbs., you would only have had to multiplied the 27 lbs. weight, just mentioned, by the ratio 3, which would have given five weights, viz., 1, 3, 9, 27, 81.

5. The number 45 may be divided into 4 such parts, that if to the first part 2 be added, from the second part 2 be subtracted, the third multiplied by 2, and the fourth divided by 2, the several results will be equal. Ans. The first, 8; second, 12; third, 5; and the fourth, 20.

6. A bought 100 animals for $100, giving $10 for oxen, $3 for swine, and 50 cents for sheep. How many of each kind did he buy?

Ans. 5 oxen, 1 hog, and 94 sheep.

7. If two of the nine figures be so placed as to count units and tens, and their sum subtracted from them, one figure of the result being known and subtracted from the figure 9, discovers the other figure of the result.

Explanation.—A asks B to think of any two figures; suppose B thinks of 5 and 3, which would read 3 units and 5 tens; then B should be required to put the two figures together, that is, call them in his own mind 53, and subtract the 5 and 3, which makes 8, from the 53, which leaves 45, and mention to you either the figure 4 or 5, just as he feels disposed, and the one mentioned taken from the figure 9, discovers the other figure.

8. A sea captain, on a voyage, had a crew of 30 men, half of whom were blacks. Being becalmed on the passage for a long time, their provisions began to fail, and the captain became satisfied that, unless the number of men was greatly diminished, all would perish of hunger before they could reach any friendly port. He therefore proposed to the sailors that they should stand in a row on deck, and that every ninth man should be thrown overboard, until one half of the crew were thus destroyed. To this they all agreed. How should they stand to save the whites?

Ans. W. W. W. W. B. B. B. B. B. W. W. B. W. W. W. B. W. B. B. W. W. B. B. B. W. B. B. W. W. B.

9. A, B and C, with their wives being on a journey, came to a river by night which it was necessary to cross. They found at the water side a boat, which could carry over but two persons at a time. Now, how could these 6 persons pass over two and two, so that none of the women should be found in company with one or two men, unless her husband were present?

*I. Two women went over, and one returned again with the boat, and repassed with the third woman.

II. One of the women went back with the boat, and stopped with her husband, while the other two men went over to their wives.

III. One of the two men, with his wife, again returned with the boat, and left his wife and carried the other man over. Then all the men and one woman were safe over together.

IV. The woman again left her husband with the other two men, and at two trips brought over the other two women, and then their navigation ended.

10. TO TELL ANY NUMBER THOUGHT OF.—Desire a person to think of any number, say his age. Request him to multiply it by three; then, if the product be an even number, divide it by 2; but if it be an odd number, add 1, and then divide it by 2. Let him multiply this quotient by 3; and if the product be even, divide it by two; if not, add one, and then divide by 2, as before. Desire him to divide the last quotient by 9, and mention to you the quotient thence arising, *regardless of the remainder*. You must multiply this quotient by 4; and if you added 1 before dividing the *first time* by 2, add one to the product; and if you added one before dividing the second time

by two, add two to this product. If the quotient cannot be divided by 9, you will add the numbers you added before dividing, calling the *first* 1, and the *second* 2, which will produce the number thought of.

11. A and B have an eight gallon cask full of cider, and wish to divide it equally, but have only two empty vessels to do it, one of which will contain five gallons, and the other three. How should they proceed?

*	8 gal. cask.	5 gal. cask.	3 gal cask.
1st.	8	0	0
2d.	3	5	0
3d.	3	2	3
4th.	6	2	0
5th.	6	0	2
6th.	1	5	2
7th.	1	4	3

Explanation—1st. The 8 gallon cask has 8 gallons in it, and the others are empty. 2d. The 8 gallon cask has 5 gallons of its contents poured into the 5 gallon cask, which fills it, and three gallons still remain in it. 3d. The 8 gallon cask still contains 3 gallons, and the 3 gallon cask is filled out of the 5 gallon one, and so on through the whole process.

12. A man owns a calf, which at the end of three years, has a female calf, also, one every year after for 20 years. Each of these calves, when 3 years old, has a calf, and also their calves, when 3 years old &c., to the end of the 20 years. Required, the owner's whole stock at the end of the 20 years.

* The increase is obtained by adding the first number to the third, this product to the second &c., throughout.

Cows, -	0	0	0	1	1	1	2	3	4	6	9	13	19	28	41	60	88	129	189	277	406
Heifers, -	0	0	1	0	0	1	1	1	2	3	4	6	9	13	19	28	41	60	88	129	189
Yearlings	0	1	0	0	1	1	1	2	3	4	6	9	13	19	28	41	60	88	129	189	277
Calves, -	1	0	0	1	1	1	2	3	4	6	9	13	19	28	41	60	88	129	189	277	406
Years, - -	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Hence, 406 Cows, 406 Calves, 277 Yearlings, and 189 Heifers.

Total, 1278, Ans.

13. *To plant a grove of* 19 *trees, having nine rows, with* 5 *trees in each row.*

RULE.

First draw a circle, and thereon
Let three *diameters* be drawn,
Which shall divide the whole in sixths,
And at each end a *dot* affix:
Then from each dot, draw, if you please,
Two *chords*, of just six scote degrees;—
Then these six lines, with those first three,
Will show you where the rows must be;
For, where your lines shall intersect,
And at said *dots* the trees erect.

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